

Exploring Magnetic Fields with a Compass

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A compass is an excellent classroom tool for the exploration of magnetic fields. Any student can tell you that a compass is used to determine which direction is north, but when paired with some basic trigonometry, the compass can be used to actually measure the strength of the magnetic field due to a nearby magnet or current-carrying wire. In this paper, we present a series of simple activities adapted from the *Matter & Interactions* textbook¹ for doing just this. Interestingly, these simple measurements are comparable to predictions made by the Bohr model of the atom. Although antiquated, Bohr's atom can lead the way to a deeper analysis of the atomic properties of magnets. Although originally developed for an introductory calculus-based course, these activities can easily be adapted for use in an algebra-based class or even at the high school level.

We performed these activities with our students in a SCALE-UP (Student Centered Active Learning Environment for Undergraduate Programs) studio at NCSU.² The most prominent aspects of this environment are the classroom layout, featuring round tables to enable face-to-face interactions, and the use of frequent, hands-on activities, such as the ones presented here.

The technique

A compass works by allowing a small magnetized needle to line up with the horizontal component of the net magnetic field in its vicinity. In most open places only the Earth's magnetic field is strong enough to significantly affect the compass needle. However, in the presence of another magnetic field of comparable strength (say, due to a magnet or a current-carrying wire), the compass needle will "deflect" from north

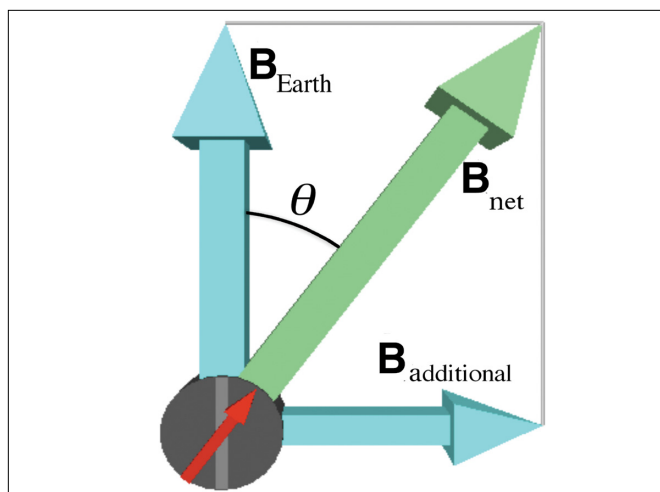


Fig. 1. The compass needle deflects by an angle θ to point in the direction of the net horizontal magnetic field.

to point in a new direction: that of the net magnetic field. We stress, of course, that the compass must be parallel with the ground so that it only responds to the horizontal components of nearby fields.

According to the superposition principle, the net magnetic field is simply the vector sum of all contributing fields:

$$\mathbf{B}_{\text{net}} = \mathbf{B}_{\text{Earth}} + \mathbf{B}_{\text{additional}} = \mathbf{B}_{\text{Earth}} + \mathbf{B}_{\text{magnet}} + \mathbf{B}_{\text{wire}} + \dots \quad (1)$$

If the situation is such that, at a particular region, this additional magnetic field is perpendicular to the horizontal component of Earth's magnetic field (Fig. 1), then the ratio of their respective strengths follows the simple trigonometric relation:

$$\frac{B_{\text{additional}}}{B_{\text{Earth}}} = \tan(\theta) \quad (2)$$

The angle θ is the deflection angle of the compass needle from the magnetic north. If both the deflection angle and the horizontal component of the Earth's magnetic field strength B_{Earth} are known, then the field strength of the additional field in the vicinity of the compass can be easily calculated. Table I shows the magnetic field strength of the Earth in a few U.S. cities. These data, along with those for other cities, can easily be found online.³

The magnetic field of a long, straight wire

For this first activity, our students used a compass to measure the magnetic field strength of a current-carrying wire. All that is needed for this experiment are two D-cell batteries, a small (~ 0.75 W) light bulb and holder, a ruler, a compass with degree markings, and a few lengths of wire, at least one

Table I. The magnetic field strength (component parallel to the ground) of the Earth at various U.S. locations.³

City	Horizontal field strength (tesla)
Anchorage, AK	1.53 e-5
Los Angeles, CA	2.47 e-5
Washington, DC	2.07 e-5
Miami, FL	2.53 e-5
Honolulu, HI	2.77 e-5
Raleigh, NC	2.2 e-5
New York, NY	2.03 e-5



Fig. 2. With no current in the wire, the compass points north.



Fig. 3. When 0.314 A is run through the wire, the compass deflects by approximately 20°.

of which should be about 30 cm long. An ammeter and a roll of tape may also be useful. Have the students prepare a simple series circuit with the light bulb and a long, straight length of wire, but leave one of the connections open for now. The light bulb should be included to reduce the current in the circuit, but it is also pedagogically useful for the students to observe that there is a correlation between the compass deflection and the bulb's illumination. Place the compass flat on a table directly beneath, and near the middle of, the straight length of wire so that the wire is lined up with the north-aligned compass needle (Fig. 2). Be sure that the compass is far from iron, steel, or other magnetic objects, especially the other parts of the circuit, since these will noticeably affect the needle. The compass and the wire may need to be taped down to reduce the problem of accidental bumps.

Once the circuit is closed, the magnetic field due to the wire will point east-west near the compass and the needle will

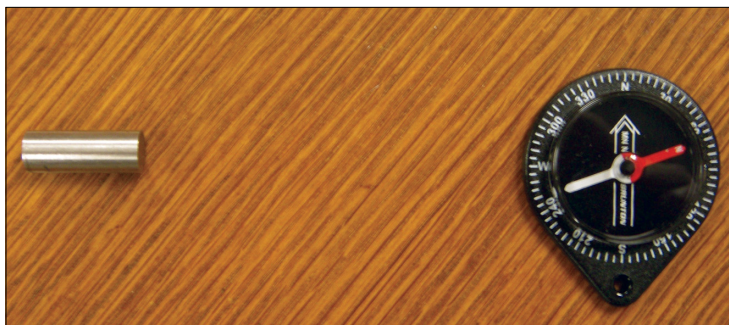


Fig. 4. The compass deflects by 70 degrees when this magnet is placed 10.8 cm (center to center distance) away.

deflect by an angle θ (Fig. 3). Using the data shown in Fig. 3 and the Table I information for Raleigh:

$$B_{\text{wire}} = B_{\text{Earth}} \cdot \tan(\theta) \\ = (2.2 \times 10^{-5} \text{ T}) \tan(20^\circ) = 8.0 \times 10^{-6} \text{ T.} \quad (3)$$

This value can be compared to that found using the Biot-Savart law for a long straight wire:

$$|B_{\text{wire}}| = \frac{\mu_0}{4\pi} \frac{2I}{r}, \quad (4)$$

where I is the conventional current in the wire, measured to be 0.314 A, and r is the distance from the center of the wire to the center of the compass needle, estimated to be 0.007 m:

$$B_{\text{wire}} = \frac{\mu_0}{4\pi} \frac{2(0.314 \text{ A})}{0.007 \text{ m}} = 9 \times 10^{-6} \text{ T.} \quad (5)$$

This turns out to be in good agreement with our measured value of the wire's magnetic field, especially considering that the precision to which our compass deflection can be measured is about $\pm 2^\circ$ and the precision of the distance estimate is, at best, ± 0.5 mm.

The magnetic field of a magnet

For the second activity, our students used this same technique to measure the magnetic field strength of a bar magnet along its axis. All that is needed is a small bar magnet and a compass. Once again, be sure that the compass is flush with the ground and is far away from other magnetic objects. Place the magnet on the "east-west" axis so that it is pointing toward the center of the compass (Fig. 4). Like the previous activity, this is set up so that, near the compass, the magnetic field due to the magnet is perpendicular to the horizontal component of Earth's magnetic field.

The magnetic field strength of the magnet can now be calculated in the vicinity of the compass:

$$B_{\text{magnet}} = B_{\text{Earth}} \cdot \tan(\theta) \\ = 2.2 \times 10^{-5} \text{ T} \cdot \tan(70^\circ) = 6.0 \times 10^{-5} \text{ T.} \quad (6)$$

Although we used a 70° deflection, this calculation is even simpler if the students adjust the separation between the compass and wire until a 45° deflection is seen. For this to

happen, the horizontal component of the Earth's field matches the magnet's field at this distance, since $\tan 45^\circ = 1$.

Distance dependence

So far, our students have only qualitatively observed that the magnetic field strength decreases the farther from the source it is measured. We encouraged our students to play with the compass and magnet to observe this effect; however, they do have the tools necessary to calculate what this distance dependence is. Mathematically, we can express this dependence as

$$|\mathbf{B}| = k(d)^n, \quad (7)$$

where k is simply a proportionality constant and n is the power relation that will be determined. It is important for the students to know that the distance dependence is different for different configurations of currents and magnets. Our students found this dependence for both the iron bar magnet, which we will focus on in this paper, and a long, straight wire.

In addition to a magnet and compass, a ruler is also needed for this activity. Carefully align the magnet as before (see Fig. 4), calculate the magnetic field strength, and measure the center-to-center distance between the compass and the magnet. Now double this distance; the magnetic field of the magnet at the new location will not be as strong. The compass deflection angle can again be used to calculate the field strength at this new location.

To calculate the factor n , take the ratio of the magnetic field strengths at these two locations:

$$\frac{B_{2d}}{B_d} = \frac{k(2d)^n}{k(d)^n} = \frac{2^n d^n}{d^n} = 2^n, \quad (8)$$

taking the natural logarithm of both sides:

$$\ln\left(\frac{B_{2d}}{B_d}\right) = \ln(2^n) = n \cdot \ln(2). \quad (9)$$

From our sample data, we calculated n to be:

$$\ln\left(\frac{6.3 \times 10^{-6} \text{ T}}{6.0 \times 10^{-5} \text{ T}}\right) / \ln(2) = -3.25. \quad (10)$$

For a magnetic dipole, the theoretical value of n should be -3 (n is -1 for a straight length of wire):

$$B_{\text{dipole, on axis}} = \frac{k}{r^3}, \quad (11)$$

where k is a proportionality constant. Again, considering the precision of both the ruler and the degree scale on the compass, this value is in good agreement with the accepted value. While this method is sufficient for making a simple comparison against the accepted value, you may want your students, depending on their abilities, to conduct a more thorough analysis by taking additional data points and constructing a log-log plot.

Looking deeper...

One of the benefits of the macro-micro connections emphasized in the *Matter & Interactions* text^{1,4} is that simple observations like the ones presented above can lead to deep analyses of physical systems. By this point, our students have seen that the field patterns for a bar magnet look remarkably like those of a single current-carrying loop of wire. They even have the same distance dependence. This leads to a direct comparison of the on-axis field equations for both situations:

$$|\mathbf{B}_{\text{loop of wire}}| = \frac{\mu_0}{4\pi} \frac{2IA}{r^3} = \frac{\mu_0}{4\pi} \frac{2\mu}{r^3}, \quad (12)$$

$$|\mathbf{B}_{\text{bar magnet}}| = \frac{k}{r^3}. \quad (13)$$

For the magnetic field equation of a loop of wire, the current in the wire and the area of the loop are the only two intrinsic properties of the loop itself. Their product, IA , is defined as the magnetic dipole moment, μ . By analogy, the magnetic field of a bar magnet can be expressed as:

$$|\mathbf{B}_{\text{bar magnet}}| = \frac{k}{r^3} = \frac{\mu_0}{4\pi} \frac{2\mu_{\text{magnet}}}{r^3}. \quad (14)$$

Using this, the students can calculate the magnetic dipole moment of their magnets. Unfortunately, it is not at all obvious where the dipole moment of a magnet originates. Is there a current in the magnet? What should the area be? These questions allow for an interesting comparison between our measurements and the predictions made by the Bohr model of the atom. It should be stressed that, while the Bohr model is only an approximation of our current understanding of the atom, it can nevertheless provide an introduction to deeper analyses.

Neils Bohr made the assumption that electrons orbit the atomic nucleus much in the same way that planets in our solar system orbit the Sun. This circular orbit can be modeled as a loop of current. If we take the electron orbit of the Bohr atom to be the primary source of the magnetic moment of an atom (neglecting the contributions from the nuclear and electronic spins),

$$\begin{aligned} \mu_{\text{atom}} &= I \cdot A = (e/T) \cdot (\pi R^2) \\ &= (ev/2\pi R)(\pi R^2) = \frac{1}{2} evR, \end{aligned} \quad (15)$$

where T is the orbital period of the electron, e is the electron charge, R is the orbital radius, and v is its orbital velocity. Compare this with the angular momentum of the electron:

$$|\mathbf{L}| = |\mathbf{R} \times \mathbf{p}| = Rmv \sin \theta = Rmv \sin(90^\circ). \quad (16)$$

Since the angular momentum L must be in integer multiples of \hbar , we now have

$$\mu_{\text{Atom}} = \frac{1}{2} evR = \frac{1}{2} \frac{e}{m} L = \frac{e}{2m} N\hbar = 9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2 \quad (17)$$

as an expression for the magnetic moment of one atom in

the ground state, for which $N=1$, as represented in the Bohr model.

As part of their investigation, our students also estimated the number of iron atoms n in each magnet so that they could compare their measurements to Bohr's prediction. Using the atomic mass of iron, along with the measured mass of one of their magnets:

$$n = (6.02 \times 10^{23} \frac{\text{atoms}}{\text{mol}}) / (55.845 \frac{\text{g}}{\text{mol}}) \cdot (13.26 \text{ g})$$

$$= 1.43 \times 10^{23} \text{ atoms.} \quad (18)$$

Assuming that these atoms are all aligned, an estimate of the dipole moment of a single atom of iron, based on our measurements, is thus:

$$\mu_{\text{Fe atom}} = \mu_{\text{magnet}} / n = \frac{(0.76 \text{ A} \cdot \text{m}^2)}{(1.43 \times 10^{23} \text{ atoms})} = 5.3 \times 10^{-24} \frac{\text{A} \cdot \text{m}^2}{\text{atom}}. \quad (19)$$

That this value is of the same order of magnitude as Eq. (17) is an excellent sign, considering the crudeness of this theoretical model.

Comparison with atomic model of iron

Of course, the currently understood model of magnetism is far more complicated than these simple calculations would suggest. Be sure to stress to your students that this analysis is based on a number of simplifying approximations to the current model.

One assumption that our analysis makes is the validity of the Bohr atom; however, the Bohr model is still widely considered to be a good first-order approximation appropriate for introductory courses. A deeper assumption made by our analysis is that the magnetic dipole moment that we measured is associated solely with the orbital angular momentum of the electron. In fact, many subatomic particles, including the electron, carry intrinsic magnetic moments associated with their spin. Any dipole moment that might be associated with an individual iron atom within a magnet arises from the complex spin-orbit interactions within the atom itself, as well as the electric interactions within large groups of atoms called *magnetic domains*. Our analysis also assumed that these domains are all precisely lined up; however, this is surely not the case. You may want to show your students some illustrations of magnetic domains to further clarify this point.

While we do not claim that this simplified analysis is correct, it does provide results that are remarkably close to those that were observed and it can be used as a palatable starting point for more advanced analyses.

Final remarks

Because they were able to observe the compass deflections rather than just read numbers off of a computer screen, our students found these experiments especially enjoyable. In addition to the experiments detailed above, our students also measured the field strength of a coil of wire. In fact, the relative simplicity of these techniques allows for their use in calculating the magnetic field strength of a wide variety of magnet and wire configurations, including using a known magnetic field to measure the field strength of Earth.^{5,6}

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