Figure 2 Solar eclipses.

the line of intersection of the lunar orbital plane and the ecliptic plane. This intersection, or line of nodes, completes one rotation in a westwardly direction every 18.6 years. Combined with the annual 365.25-day motion of the Sun from west to east around the sky and the 27.3-day orbital period of the Moon in the same direction, the potential for an eclipse exists every 173 days (the eclipse season). The actual number of eclipses visible each year varies from as few as two (both solar, as happened in 1969) to as many as seven (five solar and two lunar, which last occurred in 1935 and will occur again in the year 2160). If a lunar eclipse occurs in early January, a third lunar eclipse may occur in late December.

Various eclipse cycles have been recognized, the most famous of which is related to the saros, a period of 18 years 11 days and 8 hours that was first recognized 3,000 years ago by the Chaldeans. One saros period after the occurrence of a solar eclipse, the geometrical orientation of the Sun and Moon and the line of nodes will be similar, once again permitting the Moon to shadow the Sun, producing an eclipse; this eclipse is visible from a position 120° longitude west of the first eclipse. After three saros periods, a solar eclipse produced by a similar geometry will occur very nearly at the longitude of the original eclipse.

See also: KEPLER'S LAWS; MOON, PHASES OF; NEWTON'S LAWS; PLANETARY SYSTEMS; SUN; TIDES

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CHARLES J. PETERSON

EDDY CURRENT

See CURRENT, EDDY

EDUCATION

Because physics has a reputation for being a difficult subject, many educators are searching for ways to make it easier to learn. People working in this area

not only need to thoroughly understand the subject, but they also have to be familiar with educational methodology and psychology. Because of its multi-disciplinary nature, this subfield of physics offers a large assortment of employment opportunities. These can range from teaching in a middle school science classroom to conducting research in a university, from designing museum displays to establishing policies for federal government agencies.

As one might expect, the kind of background required for these different types of jobs varies greatly. However, the jobs can be highly satisfying and are worth investigation by people interested in promoting the understanding of physics. The most obvious choice for many people completing graduate study in physics education is to continue working in an academic setting. A teaching certificate requires a bachelor's degree, depending on the state, and opens up the middle school or high school teaching profession.

Nearly a quarter of the nation's high school seniors are enrolled in or have completed a physics course. This fraction has been steadily rising since the 1980s. This fact, coupled with an expected 20 percent rise in the number of high school age students by the year 2000, indicates that there will be a great demand for teachers. (Since a solid background in physics is a good preparation for understanding all the sciences, high school physics teachers are often also qualified to teach courses in chemistry or mathematics. For most of these teachers though, physics is their favorite subject. One survey by the American Institute of Physics found that 92 percent of teachers with multiple courses would prefer to teach physics as much as possible.) A master's degree is often the ticket to a job teaching in a community college or junior college. Two- and four-year colleges almost always have some kind of physics offering. Larger universities, which normally require faculty to hold a doctorate in their field, often have dozens of physics courses. Obviously, the more courses a school offers, the greater the variation in the topics presented and the depth of coverage.

Physics education professors often find a university combination of teaching and research duties especially stimulating. One's research into learning provides input for improving teaching performance. Conversely, being in the classroom as a teacher presents many opportunities to conduct education research. Universities also offer opportunities for outreach service to the community. Faculty members might find themselves visiting elementary school classrooms to talk about rainbows, answering telephone questions from the general public, or perhaps appearing on television to discuss the physics of a current event such as the appearance of a comet or the dangers of selling nuclear technology to unstable foreign governments.

Besides academia, there are many other areas where physics education specialists can put their skills to use. A substantial number of state and federal government agencies are involved in education-related policies. People who not only have a strong grasp of technical topics but also have insight into how people gain understanding of scientific ideas are in high demand. Additional job prospects can be found in public educational facilities like museums and in industry. A person who knows how to relate complicated technical ideas in a clear fashion might make a good marketing manager or even become a public relations expert for a high-tech company. Independent consultants can specialize in evaluating instructional programs, writing textbooks, or developing new curricular materials. As more and more computers are finding their way into homes and classrooms, the instructional software market is growing rapidly.

An interesting facet of this particular field is the strong connection it promotes between education research techniques and the concepts of physics. At the very least, good teachers apply the findings of research into how people come to grasp physical concepts. But many teachers, and especially university faculty, are getting involved in finding answers to questions about how students learn a complex subject like physics. The laboratory for this type of work can be any classroom where physics is taught at any level. Research tasks include conducting interviews of students while they think about physical situations, developing new ways of assessing knowledge and attitudes, and designing and testing innovative teaching techniques. As you might guess, one studies how people learn physics by asking a lot of questions.

Of course, these questions stem from an awareness of what other researchers have done in the past. There are thousands of articles that have been published in scientific journals about many different aspects of physics education. Searching this enormous body of literature takes time, but the process has been simplified somewhat by recent advances in database technology. It is now possible for a researcher to scan a library's entire collection from

the comfort of his or her office. Once this has been done and the papers related to the subject have been reviewed, an experimental design is developed. Often this will involve presenting students (either individually or in classes) with some sort of physical situation that focuses their attention on the concept of interest. During a series of interviews the researcher asks the students to explain what is going on in the situation in order to probe their understanding of the underlying concepts. Alternately, multiple choice testing of larger numbers of participants allows more generalizability of findings to students in other classrooms and schools. The cost of this extendability is limited resolution. (Interviews provide a much clearer picture of what an individual is thinking, but it is harder to assert that many students share those thoughts.) Many researchers are finding ways to combine these two research techniques. Qualitative interviews provide hints as to students' conceptualizations, while quantitative testing provides insight into how prevalent those ideas are in general. What we are finding is that students build their knowledge on a foundation of what they already understand about how the world works.

For the most part, these intuitive ideas are accurate; they have served their holders well for their entire lives. In a few critical cases, however, they make it difficult to learn new physics topics. A major outcome of these studies has been the realization that students can successfully complete a physics course and still have very basic misunderstandings of important topics. For example, after living their whole lives in a world where the effects of air resistance are so common that they unconsciously account for it in situations ranging from tossing out a piece of paper to playing a game of golf, students are faced with instruction that begins with "We will neglect the effects of friction." What often happens is that the connection between the real world and the artificial one presented in the classroom is never made. Although students can memorize formulas well enough to pass standard tests, they still have fundamental misconceptions. A major part of the physics education researcher's work is to find ways to assess these difficulties and then develop ways to help students come to grips with the problem. This requires a shift in thinking, a conceptual change, which is not an easy thing to do. It has been found that eliciting a student's existing concepts and then exposing them to situations conflicting with those ideas can help. Traditional instruction, where students passively sit back and are basically told the an-

swers, generally does not have a long-lasting effect on pre-existing intuitive thought patterns.

Besides investigating ways to address student misunderstanding of the basic concepts of physics, some researchers are looking at attitudes held toward physics and science in general. It is often felt that science involves nothing more than the memorization of a large number of well-established (perhaps even fossilized) facts. Real scientists, although they certainly have to know the central body of knowledge for their field of study, spend much of their time searching for answers to questions where the facts are not known. Many students are not aware of the fundamentally open-ended and non-authoritarian nature of science and that the measurements scientists make are always approximations. In other words, it is often felt that science provides absolutes that come from authority figures. Nothing could be further from the truth. Cutting-edge science is full of disagreements and scholarly debates. That is part of what makes it so fascinating.

Some physics education researchers are developing instructional activities that help students recognize how science really works, even including review of their work by their peers—the very foundation of the modern scientific enterprise. One of the advantages of being a physics education researcher is that the work can be conducted in many different settings. Because little equipment is needed, even elementary schools are potential experiment sites. A great deal of high-quality research and curriculum development has been done by high school teachers. Nonetheless, universities are where most of the investigations into how people learn physics are based.

This relatively new subfield of physics research is enjoying a period of substantial growth. During the 1980s, only two or three physics departments had graduate programs in this area. Today there are more than a dozen universities training researchers. This community of scholars meets regularly at conferences around the country. Many are invited to travel internationally, giving presentations and consulting at schools and businesses around the world. Even when these scientists are not actually in the same location, they continue to discuss ideas and share findings through heavy use of electronic mail and the World Wide Web. Collegiality is one of the greatest benefits of this type of professional endeavor.

See also: BASIC, APPLIED, and INDUSTRIAL PHYSICS; EXPERIMENTAL PHYSICS; INDUSTRY, PHYSICISTS IN

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EIGENFUNCTION AND EIGENVALUE

In many problems in physics and engineering, it is required to find a function which is not only a solution to a specified differential equation, but also has a specified behavior on the boundaries of some region of space. In mathematics, such problems are called boundary-value problems. An important class of boundary-value problems are the Sturm–Liouville problems. A Sturm–Liouville problem consists of a certain kind of second-order differential equation defined on an interval $a \leq x \leq b$ with the solutions also satisfying certain conditions to be specified below at a and at b. A Sturm–Liouville differential equation in the function $y = y(x)$ is one that can be cast into the form

$$(ry')' + qy + \lambda py = 0.$$

The prime sign indicates differentiation with respect to x. The coefficients p, q, and r are functions of x, and λ is a parameter. The only restrictions on the functions p, q, and r are that they be continuous over the interval $[a, b]$ and that r' is also continuous over that region. The Sturm–Liouville equation is very general and equations of that form occur often in physics and engineering.

There are three kinds of Sturm–Liouville problems: regular, periodic, and singular. The difference is in the form of the boundary conditions satisfied by the solutions. Regular boundary conditions are of the form

$$A_1 y(a) + A_2 y'(a) = 0$$

and

$$B_1 y(b) + B_2 y'(b) = 0,$$

where A_1 and A_2 are constants, not both zero, and B_1 and B_2 are also constants, not both zero. Periodic boundary conditions are of the form $y(a) = y(b)$ and $y'(a) = y'(b)$.

In general, solutions $y = y(x)$ may be found for a Sturm–Liouville equation for any value of the parameter λ. However, solutions that also satisfy the boundary conditions exist only for certain special values of λ. These special values are called characteristic values or eigenvalues, and are denoted λ_n. The subscript is called the index or quantum number. The solutions which correspond to particular eigenvalues are called eigenfunctions, and are labeled with the same index as the eigenvalue. That is, y_n is the function that satisfies the differential equation when $\lambda = \lambda_n$ and also satisfies the boundary conditions. While each eigenfunction corresponds to only one eigenvalue, it is possible that more than one eigenfunction may correspond to the same eigenvalue. If there are m different eigenfunctions that all correspond to the eigenvalue λ_n, that eigenvalue is said to be m-fold degenerate, and those m eigenfunctions are said to be degenerate with each other.

There are several theorems about the eigensolutions to Sturm–Liouville problems which demonstrate their importance. We list two of them here.

Theorem 1. If two eigenvalues λ_n and λ_m are distinct, then their corresponding eigenfunctions y_n and y_m are said to be orthogonal on the interval $[a, b]$ with respect to the weight function $p(x)$. This means that

$$\int_a^b p(x) \, y_m^*(x) \, y_n(x) \, dx = 0.$$

Here, the weight function $p(x)$ is the function $p(x)$ in the Sturm–Liouville differential equation and the asterisk indicates complex conjugation. This orthogonality theorem allows any function that satisfies the boundary conditions of the Sturm–Liouville problem to be expanded as a sum of eigenfunctions. The