# Deterministic Channel Modeling and Long Range Prediction of Fast Fading Mobile Radio Channels[1]

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**Abstract**

In wireless communication systems, the direct signal and the reflected signals form an interference pattern resulting in a received signal given by the sum of these components. They are distinguished by their Doppler shifts at the mobile. Once the slowly varying parameters associated with these components are determined, the fading coefficients can be accurately predicted *far ahead*. This novel approach to fading channel prediction is combined with transmitter signal optimization to mitigate the effects of "deep fades", which severely limit the performance of mobile radio systems. This capability will potentially help to reduce power requirements for wireless channels and improve the system performance.

**EDICS Number**: CL1.2.0 Wireless Techniques and Fading

[1] This research was supported by NSF grants NCR-9410227 and NCR-9726033

## 1. Introduction and Motivation

The performance of the wireless communication system is limited by intermittent power losses, or "deep fades", associated with the fading channel [1,2]. The transmission path between the transmitter and the receiver can include reflections by terrain configuration and the man-made environment. Therefore, the fading signal results from interference between several scattered signals and perhaps the direct signal [2-5]. Consider a low-pass complex model of the received signal:

$$r(t) = c(t)\, s(t) + n(t),$$

where c(t) is the flat fading coefficient (multiplicative), s(t) is the transmitted signal, and n(t) is additive white Gaussian noise (AWGN). Let the transmitted signal be $s(t)=\sum_k b_k\, g(t-kT)$, where $b_k$ is the data sequence, g(t) is the transmitter pulse shape, and T is the symbol delay. At the output of the matched filter and sampler, the discrete-time system model is given by

$$r_k = c_k\, b_k + z_k, \tag{1}$$

where $c_k$ is the complex fading signal c(t) sampled at the symbol rate, and $z_k$ is a complex discrete AWGN process with variance $N_0$. Usually, c(t) and $c_k$ are modeled as correlated complex Gaussian random processes with Rayleigh distributed amplitudes and uniform phases [1,5]. Several adaptive channel estimation methods have been developed by using this statistical description to estimate rapidly varying fading coefficients (e.g. [6]). However, the performance of these methods degrades when the fading rate increases due to a large estimation error. In addition, these algorithms do not address the most serious limiting factor in communication over fading channels. The greatest bit error rate (BER) loss and the associated high power requirements result from "deep fades" in the fading signal. Therefore, it is desirable to predict deep fades, and, in general, fading variations, and compensate for the expected power loss at the transmitter. Therefore, we address *long term* prediction of the variations in $c_k$. By prediction we imply estimating an entire future block of coefficients $c_k$ based on the observation of the received signal during an earlier time interval. This task is not feasible with current Kalman filtering and other adaptive channel estimation techniques, which can predict only one coefficient at a time, and require observation of the received sample to produce this estimate. In this paper, we propose a prediction method which would allow to determine the channel coefficients prior to transmission. In particular, the timing of future "deep fades" would be revealed and the variations in received signal power could be compensated.

## 2. Modeling and Prediction of the Fading Channel

The complex Gaussian distribution of a Rayleigh fading signal was derived based on the assumption that the scattered signals are distributed uniformly around the mobile, and that there is a continuum of scatterers [5]. Although the exact derivation of the Rayleigh fading distribution requires this assumption, it has been demonstrated that the Rayleigh fading signal can be closely approximated by a relatively small number of scatterers. For example, in the popular deterministic Jakes model, as few as nine scatterers can be used to accurately model Rayleigh fading characteristics. Moreover, physical evidence suggest that the actual number of significant scattered signals is modest (see, e.g., [3,4,7]). Therefore, we assume a channel model with a few scatterers in our numerical example, although our approach can handle as many as 1000 or more scatterers.

Due to the interference between the scattered signals, the fading signal varies rapidly for large vehicle speeds and undergoes "deep fades". From the point of view of the mobile, the fading coefficient at the receiver is given by a sum of N Doppler shifted signals

$$c(t) = \sum_{n=1}^{N} A_n \, e^{j(2\pi f_n t+\phi_n)} \tag{2}$$

where (for the $n^{th}$ scatterer) $A_n$ is the amplitude, $f_n$ is the Doppler frequency, and $\phi_n$ is the phase [5].

We predict the sampled fading signal $c_k$ (1) by decomposing it in terms of N scattered components. If the parameters $A_n$, $f_n$, and $\phi_n$ in (2) for each of the scatterers were known and remained constant, the signal could be predicted indefinitely. In practice, they vary slowly and are not known a priori. We assume that these parameters will not change significantly during any given block [4,7].

To predict the fading signals (1-2), we employ spectral estimation followed by linear prediction (LP) and interpolation [3,4]. We use the Maximum Entropy Method (MEM) for the prediction of the fast fading signal [8]. The MEM technique has the ability to fit sharp spectral features as we have in our fading channel due to the scatterers (2). Using MEM, the channel coefficients are given by the output of the autoregressive system (AR) with the transfer function:

$$H(z) = \frac{1}{1 - \sum_{j=1}^{p} d_j z^{-j}} \tag{3}$$

This model produces a current fading estimate, $\hat{c}_n$, given p past samples, $c_{n-1}...c_{n-p}$. (It might be necessary to use noisy samples as in (1) initially. This does not significantly degrade performance [3].) The $d_j$ coefficients in (3) are calculated from the poles of the transfer function given a block of channel samples (the length of this block is referred to as the observation interval). The estimates of the future samples of the fading channel can be determined as:

$$\hat{c}_n = \sum_{j=1}^{p} d_j c_{n-j} \tag{4}$$

Note that the samples have to be taken at least at the Nyquist rate, which is twice the maximum Doppler frequency, $f_{dm}$. In other approaches (e.g., [6,9]), the sampling rate used for channel estimation is the same as the data rate. However, we use much lower sampling rate. This is sufficient since the Nyquist rate is usually at least a hundred times less than the symbol rate. Therefore, given the same order p, our AR model spans much longer time interval than the model where the sampling rate is given by the data rate, and we can predict much further ahead with comparable complexity. Moreover, the accuracy of the model depends on the number of samples in the observation interval and the number of poles p. Note that p is not the same as the number of scatterers N. It is determined by the complexity constrains, and upper bounded by the length of the observation interval. It is not necessary to know N in advance, although when N is small, p can also be chosen small without degrading performance. We found that for $N<20$, $p \approx$ three to six times N assures very accurate prediction. As N increases, modest values of p are still sufficient. Even for $N=1000$, with the observation interval of 100 samples, the channel can be reliably predicted far ahead for $p=60$, although $p=99$ reduces the prediction error. Since the sampling rate for the prediction algorithm is much lower than the data rate, we employ interpolation to predict the values of the fading coefficients associated with data symbols. The interpolation method is briefly discussed in the example below.

## 3. Numerical Results and Performance Analysis

In our simulations, we assumed the maximum Doppler frequency, $f_{dm}$, is 100 Hz, and the data rate is 25 Kbps. We sample the channel at the rate of 500 Hz. Thus, there are 50 data points between adjacent sampling points. To determine initial observations of the fading coefficients, $c_k$, at the sampling points, one can send training symbols $b_k$ at the channel sampling rate of 500 Hz (see (1)). This overhead affects the throughput only by 2%. In Figure 1, we examine the Jakes channel model with nine oscillators (scatterers) [5], i.e., N=9 in (2). The channel is observed for the first 100 samples (0.2 seconds). Here actual channel coefficients are used during observation. As shown in [3], noisy measurements do not significantly increase the prediction error. In this example, we chose p=60 in (4). By employing MEM, the prediction coefficients $d_j$ in (4) were determined. Since actual channel coefficients are not available beyond the observation interval, the estimates of previous p fading coefficients are used to form future predicted values,. For example, earlier predicted values $\hat{c}_{n-j}$ can be used instead of the actual values $c_{n-j}$ in (4) to form future estimates $\hat{c}_n$. This approach was taken in [3]. However, it was observed that this method causes error propagation later in the prediction. Therefore, we are investigating adaptive algorithms to update channel estimates during transmission. Using adaptive channel estimation combined with transmitter pre-compensation (see below), more reliable data-aided estimates of fading coefficients can be obtained at the receiver, and fed back to the transmitter at the sampling rate. Our results (not shown here due to space constraints) indicate that this technique significantly reduces error propagation, and that the channel can be accurately forecasted for several hundred of future data symbols. Therefore, in our performance analysis below we are assuming that perfect estimates of fading coefficients $c_{n-j}$ are available in (4), and in Figure 1 we use actual fading coefficients for prediction. This assumption is realistic in view of the accuracy of the combined prediction and tracking technique.

The future values of the channel coefficients are predicted and plotted in dotted lines for the last 100 samples in the Figure 1. It can be seen that the predicted values follow very closely the actual future envelope shown in solid lines. Therefore, we can determine future channel variations and predict when the channel is going to enter deep fades in the future.

Note that Figure 1 shows only sampled points at the rate of 500 Hz. Since this sampling rate is much lower than the data rate, we perform interpolation between predicted channel coefficients to get better resolution. In this interpolation process, four consecutively predicted channel coefficients are interpolated by a Raised Cosine (RC) filter to generate estimates of the fading coefficients, $\hat{c}_k$, between two adjacent predicted samples at the data rate [10]. We found that for the normalized sampling rate $f_s'=2.5$, where $f_s'$ is given by $f_s/2f_{dm}$ , the optimum rolloff factor is 0.64. Although $f_s'=2.5$ results in oversampling, it produces much more accurate interpolated values than lower values of $f_s'$.

The proposed prediction method can be combined with tracking and transmitter optimization. In our simulations, we assumed coherent detection and used Binary Phase Shift Keying (BPSK) modulation scheme. Given binary signal $b_k$ and $E(|c_k|^2) = 1$, the signal-to-noise (SNR) is $\gamma_b = \frac{E(b_k^2)}{N_0}$. The following channel inversion with threshold method is investigated to accomplish reliable communication. The channel samples taken during the observation interval are sent to the transmitter, which applies MEM and adaptive linear prediction, and interpolates to produce predicted fading values at the data rate. Note that this feedback is not going to introduce significant delay since the sampling rate is much lower than the data rate. The transmitter interrupts the transmission for the k-th symbol if the power level, $|\hat{c}_k|^2$, is below previously chosen threshold value. Furthermore, if $|\hat{c}_k|^2$ is above the threshold, the transmitter sends the data bits, $b_k$, by multiplying them with the inverse of the predicted $\hat{c}_k$ values (4). This power adjustment is not proposed as a practical solution, since it will result in large transmitter power fluctuations. It is considered here to access performance advantages of the proposed prediction technique. We are currently investigating efficient adaptive coding and modulation methods for transmitter optimization [11]. The bit error rates (BER) for this channel inversion with threshold method for the nine-oscillator model and the prediction algorithm described earlier in this section are plotted in Figure 2. With no threshold and no compensation, the channel exhibits Rayleigh fading characteristics, and the bit error rate (BER) is [1]

$$P_e = \frac{1}{2}\left(1-\sqrt{\frac{\gamma_b}{1+\gamma_b}}\right) \tag{5}$$

By increasing the threshold from 0.1 to 0.6, we observe performance improvement. However, the throughput reduces with the increasing thresholds (or equivalently, the bandwidth increases). The throughputs are 90.5%, 82%, 67%, and 55% for the thresholds 0.1, 0.2, 0.4, and 0.6 respectively. The simulation results slightly deviate from the theoretical values due to the prediction and the interpolation errors. Since the power of the transmitted signal $\frac{b_k}{\hat{c}_k}$ is greater than $E(b_k^2)$ for thresholds <0.4, the BER for these threshold values are above the AWGN channel BER. For the threshold=0.4, the transmitted power is equal to $E(b_k^2)$, and the analytical curve is also the BER of the AWGN channel [1]

$$P_e = Q(\sqrt{2\gamma_b}) \tag{6}$$

where the $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} \exp(-t^2/2)dt$. Thus, by using the proposed prediction method, we were able to reduce the BER to and beyond the level of the AWGN channel.

## 4. Conclusions and Future Work

The flat fading channel is characterized in terms of the characteristics of important scatterers, and the fading prediction and precompensation method is proposed. Initial results show the potential for reliable low power wireless communication. Several important questions related to the proposed method are addressed in our current work. We are studying the extension of the proposed method to differentially encoded signals, multipath fading channel and antenna arrays. We are also addressing joint prediction and adaptive coding and continuing realistic channel modeling necessary to verify feasibility of the proposed method.


## Acknowledgment

The authors are grateful to Brian Hughes for his suggestions and helpful comments, and to Shengquan Hu for generating adaptive prediction and tracking results.


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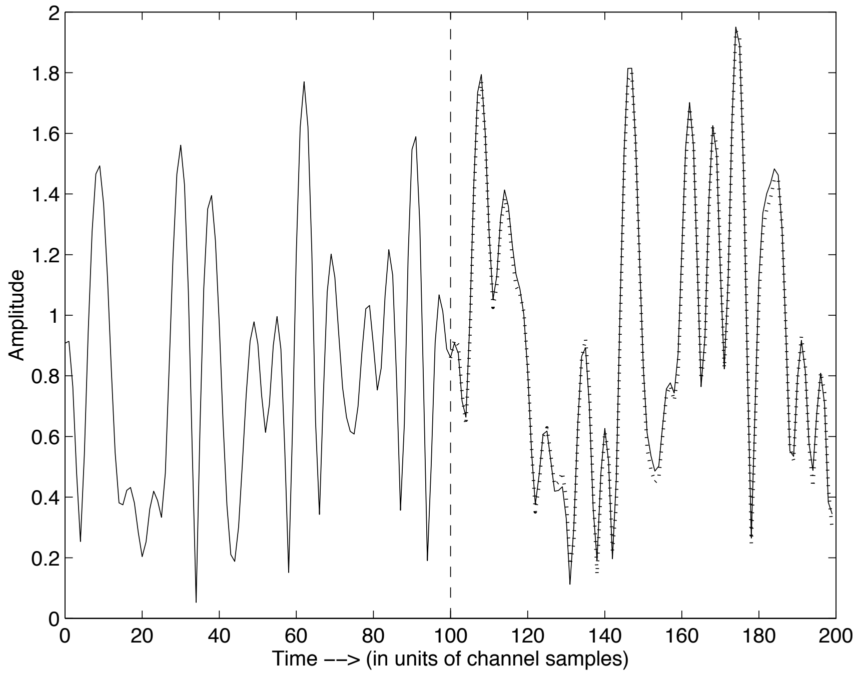


**Figure 1**: First half: The actual fading channel envelope (solid line) is observed. Second half: The actual future (solid line) and predicted (dotted line) fading channel envelopes for the maximum Doppler frequency, $f_{dm}$ =100 Hz.

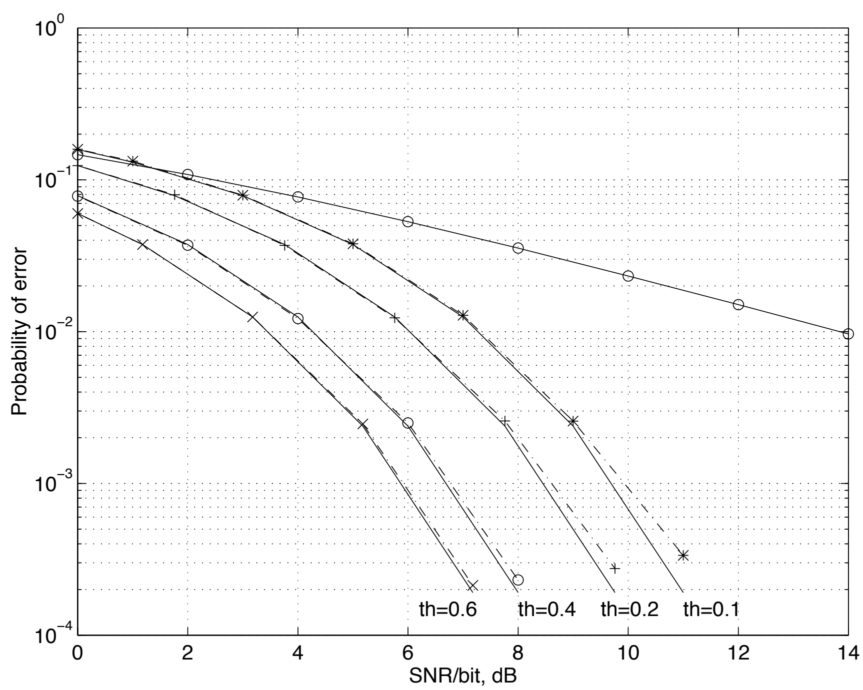


**Figure 2**: Probability of bit error vs SNR for Rayleigh fading channel with no threshold and no compensation at the transmitter (o—–o), with thresholds and compensation at the transmitter for thresholds 0.1 (*----*), 0.2 (+----+), 0.4 (o----o) (also Gaussian channel BER), and 0.6 (x----x). The dashed lines are the simulations and the solid lines are the theoretical results for the each threshold level.