SUBCHANNEL ALLOCATION FOR MULTICARRIER CDMA WITH ADAPTIVE FREQUENCY HOPPING AND DECORRELATING DETECTION

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ABSTRACT

Multicarrier Code-Division Multiple Access (MC-CDMA) system with adaptive frequency hopping (FH) has attracted significant attention in the literature due to its excellent spectral efficiency. A suboptimal water-filling (WF) channel allocation algorithm was previously proposed for the conventional matched filter (MF) detector in the reverse link of this system. However, the performance of the WF algorithm is degraded by the fading-induced near-far problem. We propose a new allocation algorithm to overcome the limitations of the WF algorithm, and demonstrate the resulting BER improvement using simulation. Moreover, we employ the linear decorrelating detector at the receiver of the MC-CDMA system with adaptive FH to improve the spectral efficiency. The proposed allocation algorithm is also extended to this receiver by exploiting the SNR analysis of decorrelator. We demonstrate that the linear decorrelating detector that employs the proposed allocation algorithm is very effective in mitigating MAI, with performance approaching the single user bound for MC-CDMA system with adaptive FH.

I. INTRODUCTION

Several MC-CDMA systems with adaptive frequency hopping (FH) were proposed for the cellular communication systems in [1,3,4] and have attracted significant attention due to their excellent spectral efficiency. In these systems, the data of each user is multiplexed over one or several substreams, and multiple subcarriers are employed to transmit the substreams of all users. Direct sequence spread spectrum (DS/SS) codes are assigned to all substreams in the system. Each substream is transmitted on one subcarrier. The subcarriers are allocated at the transmitter using the knowledge of the channel state information (CSI) fed back from the receiver. MC-CDMA with adaptive FH exploits both frequency and multiuser diversity and improves on non-adaptive MC-CDMA systems [2].

In [3], a MC-CDMA system with adaptive frequency hopping was proposed for the forward link. In this system, one substream per user is employed, and each user selects the subcarrier with the largest fading amplitude for transmission. This allocation algorithm provides significant performance improvement over other diversity techniques such as MC-CDMA with the maximum ratio combining (MRC) [2]. However, this simple allocation algorithm does not take into account the multiple-access interference (MAI) present in realistic wireless systems. A near-optimal allocation algorithm was utilized in [5] to maximize the total average signal to interference and noise ratio (SINR). This method improves on the performance of the simple allocation policy in [3].

For the reverse link, a MC-CDMA system with adaptive FH was investigated in [1]. In this system, multiple substreams are employed for each user, and random signature sequences are assigned for all substreams, resulting in enhanced MAI and intra-user interference. A sub-optimal water-filling (WF) allocation algorithm was proposed, and it was demonstrated this system has better performance than single carrier DS-CDMA system with RAKE receiver and the conventional MC-CDMA system [1]. However, the WF algorithm in [1] offers limited protection to weaker users. Thus, it suffers from the near-far problem that is caused by the short term fading. As a result, the WF algorithm has high Bit Error Rate (BER) in moderate to high SNR region.

In this paper, we also investigate MC-CDMA system with adaptive FH for the reverse link. The major difference between this system and that in [1] is utilization of orthogonal spreading sequences for all substreams of the same user, resulting in reduced MAI. We propose a new allocation algorithm to overcome the limitations of the WF algorithm in [1], and demonstrate the resulting BER improvement using analysis and simulations.

In [1], the conventional matched filter receiver was employed at the receiver of the MC-CDMA systems with adaptive FH, and the performance was limited by MAI.

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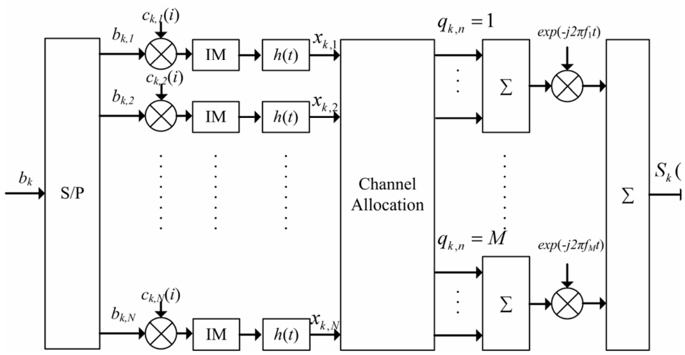

Fig. 1 Transmitter of MC-CDMA system with adaptive FH at the mobile for the k^{th} user.

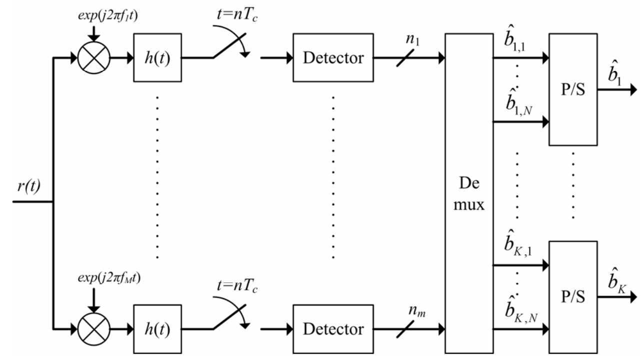

Fig. 2 Receiver structure of MC-CDMA system with adaptive FH hopping at the base station.

While it has been demonstrated that Multiuser Detection (MUD) can greatly improve spectral efficiency of CDMA systems [6], to the best of our knowledge, utilization of MUD and its impact on the adaptive allocation algorithm has not been not reported for MC-CDMA with adaptive FH. In this paper, we employ the linear decorrelating detector at the receiver of the proposed MC-CDMA system. The proposed allocation algorithm is extended to this receiver by exploiting the signal to noise ratio (SNR) analysis of decorrelator [1]. To reduce complexity of this allocation method, recursive update of the matrix inverse is utilized [7]. Performance gain achieved using the decorrelator combined with the proposed allocation method is demonstrated.

The remainder of this paper is organized as follows. In section II, we describe the system model of the proposed MC-CDMA system with adaptive FH. In Section III, the limitations of the WF algorithm are discussed, and the proposed allocation algorithm is described. In Section IV, the proposed allocation algorithm for the decorrelating detector is described. Numerical results and conclusion are contained in Sections V and VI, respectively.

II. SYSTEM MODEL

A. Transmitter

In the multicarrier CDMA system, the total bandwidth W is divided into M subcarriers with equal bandwidths W/M. We consider the reverse link transmission. The data stream of each user is multiplexed over N substreams, and all substreams are spread by spreading codes in time domain. Then the spread signal is passed through the impulse modulator (IM) and a chip wave-shaping filter. The resulting equivalent low-pass signal for the n^{th} substream of k^{th} user can be expressed as

$$x_{k,n}(t)=\sqrt{2E_b}\sum_{l=-\infty}^{\infty}\sum_{i=0}^{PG-1} b_{k,n}(l)c_{k,n}(i)h(t-lT_b-iT_c) \qquad (1)$$

where E_b is the bit energy, T_c is the chip duration, T_b is the bit duration, $PG=T_b/T_c$ is the processing gain, and the vector $\mathbf{c}_{k,n}=[c_{k,n}(0)\ \ c_{k,n}(2)\ \ \ldots\ \ c_{k,n}(PG\text{-}1)]^T$ denotes the spreading code normalized as $\|\mathbf{c}_{k,n}\|=1$, where $\|*\|$ is the Euclidean norm. The information bit $b_{k,n}$ takes on the values in the set {+1,-1} since we assume binary phase-shift keying (BPSK) modulation. The impulse response of the chip wave-shaping filter $h(t)$ satisfies the Nyquist criterion to avoid inter-chip interference[8]. It is normalized as [2]

$$\int_{-\infty}^{\infty}|H(f)|^2\, df=1,$$

where $H(f)$=F[$h(t)$] is the Fourier transform of $h(t)$.

After spreading, each substream is assigned to one of the M subcarriers by a control unit at the base-station. In this paper, we use idealized assumption that the perfect CSI, timing and frequency offsets of all users are known at the base-station. The algorithm design and performance analysis for realistic systems is the subject of future investigation. Using an allocation method, the base-station sends the transmitting subcarrier frequencies to the mobiles using a forward control channel. The allocation algorithm assigns the n^{th} substream of the k^{th} user to the $q_{k,n}^{\text{th}}$ subcarrier, where $n\in[1,N]$, $k\in[1,K]$, and $q_{k,n}\in[1,M]$. As in [1], more than one substream of the same user can hop onto the same subcarrier. The total number of substreams on the m^{th} subcarrier is

$$n_m=\sum_{k=1}^{K}\sum_{n=1}^{N}\delta(q_{k,n}\text{-}m) \qquad (2)$$

where

$$\delta(x)=\begin{cases}1, & x=0\\ 0, & \text{otherwise}\end{cases}$$

While the best performance results when the spreading codes of all substreams are orthogonal, this condition is impractical due to two reasons. The first reason is that the number of available orthogonal spreading codes is usually less than the total number of substreams [1]. For example, we used a system with 128 substreams and a processing gain of 64 in our simulations, and orthogonal codes cannot be used in this case. Second, ideal synchronization (i.e., exact timing and carrier frequency alignment for different users) is difficult to achieve in practice. Therefore, orthogonal codes may result in large cross-correlations when synchronization is not perfect [9]. As discussed in [9], quasi-synchronous assumption is suitable for the reversed link in practice, i.e., the timing of all users is aligned within in a small synchronization window. This assumption is used in this paper to model MAI due to non-orthogonal signature sequences associated with different users. However, orthogonality can be maintained between the substreams of the same user, i.e.

$$\mathbf{c}_{k,n}^{T}\mathbf{c}_{k,n'}=0 \text{ if } n'\neq n, \quad (3)$$

The condition (3) is utilized in this paper to eliminate self-interference. In this paper, all users are assigned random spreading sequences with

$$\mathrm{E}(\mathbf{c}_{k,n}^{T}\mathbf{c}_{k',n'})=0 \text{ and } \mathrm{E}[(\mathbf{c}_{k,n}^{T}\mathbf{c}_{k',n'})^{2}]=\frac{1}{PG}, \text{ if } k'\neq k, \quad (4)$$

and a set of N orthogonal Walsh codes is used to provide orthogonality (3) between the substreams of the same user [10]. The signature sequence of each substream is the product of user specific spreading code and one of the Walsh codes.

Finally, the low-pass equivalent transmitted signal for k^{th} user is the sum of modulated substream signals

$$S_k(t)=\sum_{n=1}^{N} x_{k,n}(t)\exp(-j2\pi f_{k,q_{k,n}}t) \quad (5)$$

where $f_{k,q_{k,n}}$ is the subcarrier frequency offset from carrier frequency f_c for $q_{k,n}{}^{\text{th}}$ subcarrier of the k^{th} user. The complete transmitter structure of this system is shown in Fig. 1.

B. Channel and Receiver

We assume slowly varying Rayleigh fading, i.e., the fading coefficients do not change over several consecutive symbols. Moreover, it is assumed that the signal transmitted over each subcarrier experiences flat fading, and the fading coefficients of different subcarriers are statistically independent [2]. The complex channel coefficient of subcarrier m of the k^{th} user can be expressed as

$$\gamma_{k,m}=\alpha_{k,m}\mathrm{e}^{-j\phi_{k,m}}. \quad (6)$$

where the amplitude of the channel gain $\alpha_{k,m}$ has the probability density function

$$p(\alpha_{k,m})=\alpha_{k,m}\, e^{-\alpha_{k,m}^{2}/2},\ \alpha_{k,m}\geq 0 \quad (7)$$

and the phase $\phi_{k,m}$ is uniformly distributed on the interval [0, 2π].

The structure of the receiver is shown in Fig. 2. The low-pass equivalent received signal at the base station is

$$r(t)=\sum_{k=1}^{K}\sum_{n=1}^{N}\gamma_{k,q_{k,n}}\, x_{k,n}(t-\tau_k)\exp\left[-j2\pi f_{k,q_{k,n}}(t-\tau_k)\right]+n(t) \quad (8)$$

where $n(t)$ is the complex additive white noise with power spectral density N_0 and τ_k is the timing offset of the k^{th} user. Define U_{m} as the ordered index set of all substreams that share the m^{th} subcarrier, i.e.

$$U_m=\{(k,n)\mid q_{k,n}=m\} \quad (9)$$

The size of U_{m} is denoted as n_m, and the ordered pair (k,n) is used to refer to the corresponding substream in (9) through this paper. Let $(k,n)\in U_m$. Without loss of generality, assume $\phi_{k,m}=0$ and $\tau_k=0$. The output of the chip-matched filter corresponding to this substream is

$$y_{k,n}(t)=S_{k,n}(t)+I_{k,n}(t)+n_{k,n}(t) \quad (10)$$

In this expression, the signal of the desired substream is

$$S_{k,n}(t)=\sqrt{2E_b}\,\alpha_{k,m}\sum_{l=-\infty}^{\infty}\sum_{i=0}^{PG-1} b_{k,n}(l)c_{k,n}(i)\chi(t-lT_b-iT_c)$$

where $\chi(t)$ is the inverse Fourier transform of $|H(f)|^2$, i.e., the autocorrelation function of $h(t)$. The interference from other subcarriers is negligible due to the frequency separation and spreading [1,2]. The inter-substream interference for given subcarrier can be expressed as

$$I_{k,n}(t)=\sum_{\substack{(k',n')\in U_m\\(k',n')\neq(k,n)}}\exp\left\{-j\phi_{k',m}+j2\pi\left[(f_{k,m}-f_{k',m})t+\tau_{k'}\right]\right\}S_{k',n'}(t-\tau_{k'}).$$

Finally, $n_{k,n}(t)$ is the filtered noise term. The sampled output of the matched filter is correlated with the local spreading sequence reference $\mathbf{c}_{k,n}$. Without loss of generality, we consider only one output symbol of this correlator and drop the symbol index l. This output is given by [1,2]

$$Z_{k,n}=\hat{S}_{k,n}+\hat{I}_{k,n}+\hat{N}_{k,n} \quad (11)$$

where $\hat{S}_{k,n}=\sqrt{E_b}\,\alpha_{k,m}b_{k,n}$ carries the desired bit information and $\hat{I}_{k,n}$ is the interference term

$$\hat{I}_{k,n}=\sqrt{E_b}\sum_{\substack{(k',n')\in U_m\\(k',n')\neq(k,n)}}\alpha_{k,m}\exp\left[-j\phi_{k',m}\right]b_{k',n'}\rho_{(k',n')(k,n)}$$

where

$$\rho_{(k',n')(k,n)}=\sum_{i=0}^{PG-1}c_{k',n'}(i)c_{k,n}(i)e^{j2\pi\left[(f_{k,m}-f_{k'm})iT_c+\tau_{k'}\right]}\chi(-\tau_{k'})$$

is the cross-correlation between the waveforms of corresponding substreams. Since the reverse channel is quasi-synchronous, it is reasonable to ignore the interference from adjacent symbols in (11). The second moment of the zero mean Gaussian noise term $\hat{N}_{k,n}$ in (11) is $E(\hat{N}_{k,n}\hat{N}^{*}_{k',n'})=N_0\,\rho_{(k',n')(k,n)}$ for $(k,n)\in U_m$, $(k',n')\in U_m$.

III. ALLOCATION ALGORITHMS FOR MATCHED FILTER RECEIVER

A. The Waterfilling algorithm

When the number of users K is large, the interference $\hat{I}_{k,n}$ can be modeled as Gaussian noise with the variance that depends on the channel conditions. The average variance of $\hat{I}_{k,n}$ is

$$\mathrm{Var}(\hat{I}_{k,n})=\frac{\beta E_b}{PG}\sum_{(k',n')\in U_m\ \&\ k'\neq k}\alpha_{k',m}^2 \qquad (12)$$

where β depends on the delay spread of the fading channel T_d, chip period T_c, as well as timing and frequency synchronization. Assuming perfect synchronization and $T_d/T_c=1$, the value of $\beta=1$ [1].

For the matched filter receiver [1], the BER of the substream (k,n) can be approximated as

$$P_e(k,n)\approx \mathrm{Q}\left(\sqrt{2\lambda(k,n)}\right) \qquad (13)$$

where $\mathrm{Q}(x)=\frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-t^2/2)dt$, $x\geq 0$ and $\lambda(k,n)$ is the SINR of substream (k,n) that can be calculated as

$$\lambda(k,n)=\frac{\hat{S}_{k,n}^2}{\mathrm{Var}(\hat{I}_{k,n})+\mathrm{Var}(\hat{N}_{k,n})}=\frac{E_b\alpha_{k,m}^2}{\frac{\beta E_b}{PG}\sum_{\substack{(k',n')\in U_m\\ k'\neq k}}\alpha_{k',m}^2+N_0} \qquad (14)$$

The SINR (14) can be used by subcarrier allocation algorithms to exploit both frequency and multiuser diversity. The water-filling scheme proposed in [1] is an iterative allocation algorithm. One substream is allocated in each iteration. The sequence of KN iterations is divided into N consecutive groups with K iterations per group. In each group, one substream per user is allocated. During each iteration of group n, all users whose substreams have not yet been allocated in this group select the subcarrier with the highest SINR (14), and the user that has the lowest selected SINR level is assigned to that subcarrier [1]. The protection of weak users offered by the order of subcarrier allocation in the WF is limited due to two reasons. First, weak users already assigned to given subcarrier cannot contribute large interference level that would prevent allocation of a much stronger user's substream to this subcarrier. Second, the WF algorithm does not take into account the MAI impact of assigning a new substream on the substreams already allocated to certain subcarrier. As a result, the BER of the WF allocation algorithm is very high for some realizations of channel fading coefficients. This case is demonstrated in the following example.

Example Consider a system with $K=N=M=2$, $PG=16$, $E_b=1$, $N_0=0.01$, and $\beta=1$. Random codes are assigned to substreams of different users, while the substreams of each user are orthogonal as described in Section II. The fading amplitudes are represented by the matrix:

$$\begin{bmatrix}\alpha_{1,1}^2 & \alpha_{1,2}^2\\ \alpha_{2,1}^2 & \alpha_{2,2}^2\end{bmatrix}=\begin{bmatrix}0.1 & 0.09\\ 1 & 0.1\end{bmatrix}$$

The WF algorithm [1] will perform the allocation in four iterations. The resulting allocation is:

$$\begin{bmatrix}q_{1,1} & q_{1,2}\\ q_{2,1} & q_{2,2}\end{bmatrix}=\begin{bmatrix}1 & 2\\ 1 & 1\end{bmatrix}$$

Using Gaussian approximation, we obtain the BER of substream (1,1) as

$$P_e(1,1)\approx \mathrm{Q}\left(\sqrt{\frac{2\times 0.1}{0.01+2/16}}\right)\approx 0.11$$

A better allocation solution is to assign the substreams of the first user to the first subcarrier and the substreams of the other user to the second subcarrier. The resulting BERs are:

$$P_e(i,j)\approx \mathrm{Q}\left(\sqrt{\frac{2\times 0.1}{0.01}}\right)\approx 3.8\times 10^{-6}\ \text{ for } i,j\in\{1,2\}$$

This example illustrates that the WF algorithm has poor performance when the fading amplitude of one user is much larger than that of the other user. While such events are unlikely, they dominate the average BER in moderate to high SNR region. Therefore, it is desirable to design an allocation algorithm that overcomes the limitations of the WF method.

B. The proposed algorithm

As in [1], we use SINR instead of BER as the performance measure, and focus on the performance of the substream with the lowest SINR since the error events associated with this substream dominate the error rate. The optimization objective is to maximize the SINR of the allocated substream with the lowest SINR, i. e.

$$Q_o=\arg\max_{Q}\left\{\min_{\substack{k'\in[1,K]\\ n'\in[1,N]}}\lambda(k',n')\right\} \qquad (15)$$

where $Q=\{q_{k',n'}|k'\in[1,K],n'\in[1,N]\}$ is the set of all allocation variables.

Theoretically, the search over the elements Q produces the optimal solution for (15). For one user, there are $N_w=(N+M-1)!/[(M-1)!N!]$ possible ways to allocate its N equivalent substreams into M distinct subcarriers. For K users, the number of possible ways is N_w^K and the computational complexity of finding the exact solution of (15) is not affordable. Thus, a suboptimal method with moderate complexity is desirable.

Inspired by the iterative algorithms in [1,5], we design an iterative allocation method to find a suboptimal solution for (15). In this algorithm, one substream is assigned at each iteration, and all substreams are allocated after KN iterations. For simplicity, the substreams are assigned consecutively, i.e. the substream (k,n) is allocated at the $[K(n\text{-}1)+k]^{th}$ iteration. This assignment order results in negligible performance degradation relative to more sophisticated methods.

Define the subset U that includes the substreams that has been already assigned and the substream (k,n), i.e. $U=\{(k',n')|\ K(n'\text{-}1)+k'\leq K(n\text{-}1)+k\}$. When (k,n) is allocated to the subcarrier $q_{k,n}$, only the elements of U that are assigned to this subcarrier experience the SINR degradation. Our goal is to reduce the degradation associated with allocating a new substream (k,n). To achieve this goal, we determine the lowest SINR level achieved if (k,n) is assigned to the subcarrier $q_{k,n}$, where there are M possible choices for $q_{k,n}$. Then we allocate (k,n) to the subcarrier that produces the maximum value of this lowest SINR among all subcarrier allocation choices. Thus, the allocation rule is:

$$q_{k,n}=\arg\max_{q_{k,n}}\left\{\min_{\substack{(k',n')\in U\\ \&\ q_{k',n'}=q_{k,n}}}\{\lambda'(k',n')|Q'\}\right\} \tag{16}$$

where $\lambda'(k',n')$ is the SINR level of the substream (k',n') when only the interference from the subset U is taken into account, and the set of all prior allocations is $Q'=\{q_{k',n'}|\ K(n'\text{-}1)+k'<K(n\text{-}1)+k\}$.

In summary, our allocation algorithm consists of the following steps:

S0) Initialize $n_m=0$, $k=1$, $n=1$, $A_m=\{\emptyset\}$ and $U_m=\{\emptyset\}$ for all $m\in[1,M]$, where n_m is the number of substreams, and A_m and U_m are the sets of channel coefficients and substream indexes associated with the m^{th} subcarrier, respectively.

For $n=1\ldots N$,

For $k=1\ldots K$,

S1) Define augmented sets $A'_m=\{A_m,\alpha_{k,m}\}$ and $U'_m=\{U_m,\ (k,n)\}$ for $m\in[1,M]$. The elements in A'_{m} and U'_m are indexed as $A'_m(p)$ and $U'_m(p)$, respectively, where $p\in[1\ldots n_m+1]$. The element $A'_m(p)$ is the fading coefficient (6) of the substream $U'_m(p)$. Then calculate $SINR_{m,p}$ defined in the equation (18) below**.** The $SINR_{m,p}$ is the SINR of the substream $U'_m(p)$ if the substream (k,n) is assigned to the m^{th} subcarrier.

S2) Find m_o that satisfies

$$m_o=\arg\max_{m\in[1,M]}\left\{\min_{p\in[1,n_m+1]}(SINR_{m,p})\right\} \tag{17}$$

S3) Assign substream (k,n) to the m_o^{th} subcarrier, i.e. set $q_{k,n}=m_o$. Then update $n_{m_o}=n_{m_o}+1$, $A_{m_o}=\{A_{m_o},\alpha_{k,m_o}\}$, and $U_{m_o}=\{U_{m_o},\ (k,n)\}$.

The SINR level at the output of the matched filter receiver is given by (14), and the allocation algorithm uses $\lambda'(k',n')$ described after equation (16). Therefore, $SINR_{m,p}$ in S1 can be computed as

$$SINR_{m,p}=\frac{E_b|A'_m(p)|^2}{\frac{\beta E_b}{PG}\sum_{i=1}^{n_m+1}\Delta\{U'_m(i),U'_m(p)\}|A'_m(i)|^2+N_0} \tag{18}$$

where

$$\Delta\{(k_1,n_1),(k_2,n_2)\}=\begin{cases}1, \text{ if } k_1\neq k_2\\ 0, \text{ otherwise}\end{cases}$$

IV. ALLOCATION ALGORITHM FOR DECORRELATING DETECTOR

When the number of users in the system is large, the performance of MC-CDMA system will be limited by the strong MAI [1,3]. Multiuser detection techniques can be employed to suppress MAI. The proposed MC-CDMA system employs DS/SS. We investigate utilization of existing MUD techniques for DS CDMA [6] to reduce MAI for each subcarrier. The objective is to improve the spectral efficiency of the MC-CDMA systems with adaptive FH. The complexity of the optimal MUD prohibits its implementation in our system. In this paper, we employ a suboptimal MUD, the linear decorrelating detector, that provides good performance-complexity trade-off in CDMA systems [6]**.**

Consider the outputs of the matched filters for the $P=n_m$ substreams allocated to the m^{th} subcarrier. Without loss of generality, replace the substream index pair (k,n) in (11) with a single index $p\in[1,P]$. Suppose the decorrelating detector is applied to the P substreams allocated to the subcarrier m. In the matrix form, the outputs of the matched filters are rewritten as [6]

$$\mathbf{z}=\mathbf{RAb}+\hat{\mathbf{n}} \tag{19}$$

where $\mathbf{z}=\left[Z_1,Z_2,\ldots,Z_P\right]^T$, the $P\times P$ autocorrelation matrix $\mathbf{R}$ with components $\mathbf{R}_{i,j}=\rho_{i,j}$, $\mathbf{A}=\text{diag}\{\gamma_1,\gamma_2,\ldots,\gamma_P\}$, $\mathbf{b}=\left[b_1,b_2,\ldots,b_P\right]^T$, and $\hat{\mathbf{n}}=\left[\hat{N}_1,\hat{N}_2,\ldots,\hat{N}_P\right]^T$. In above expressions, Z_p, b_p, γ_p, and $\hat{N}_p$ are the output of the correlator, the information bit, the channel coefficient, and the noise component for the p^{th} substream, respectively. The cross-correlation between the waveforms of i^{th} and j^{th} substream is $\rho_{i,j}$ (see (11)). The covariance matrix of $\hat{\mathbf{n}}$ is $N_0\mathbf{R}$.

The output of the decorrelating detector is [6]

$$\mathbf{R}^{-1}\mathbf{z}=\mathbf{Ab}+\mathbf{R}^{-1}\hat{\mathbf{n}} \tag{20}$$

For BPSK modulation, the demodulation is performed as

$$\hat{\mathbf{b}} = \text{sgn}\{\text{Re}(\mathbf{A}^H\mathbf{R}^{-1}\mathbf{z})\}, \quad (21)$$

where sgn() is the sign function, $\hat{\mathbf{b}}$ is the decision vector, and $\mathbf{A}^H$ is the Hermitian transpose of matrix **A**. The SNR of the decorrelator for the p^{th} substream on the m^{th} subcarrier is [6]

$$SNR_{m,p} = \frac{|\gamma_p|^2}{N_0\mathbf{R}^{-1}(p,p)} \quad (22)$$

where $\mathbf{R}^{-1}(i,j)$ denotes $(i,j)^{\text{th}}$ element of matrix $\mathbf{R}^{-1}$.

Now, we describe the subcarrier allocation method for the case when the decorrelating detector is employed at the receiver. Since the decorrelating detector completely eliminates MAI, the SINR has to be replaced by the SNR as the performance measure. However, since the proposed allocation method for the decorrelator follows the same steps as the algorithm proposed in Section III for the matched filter, we retain the same notation in the algorithm description, but use the expression (22) instead of (18) when calculating the SINR in (17). In particular, we employ in S1:

$$SINR_{m,p} = \frac{|A'_m(p)|^2}{N_0\mathbf{R}_m^{-1}(p,p)} \quad (23)$$

where $(n_m+1)\times(n_m+1)$ matrix $\mathbf{R}_m$ is the correlation matrix of substreams in set U'_m, i.e. $R_m(i,j)=\rho_{U_m(i),U_m(j)}$.

This allocation algorithm is very complex due to the matrix inversion in (23). However, it is possible to reduce complexity by recursive computation of the inverse. In this recursion, the result of previous iteration is utilized as follows. Suppose that P substreams have been previously allocated to a subcarrier. Denote the $P\times P$ autocorrelation matrix of these substreams as $\mathbf{R}(P)$. The inverse of this matrix has been computed in the previous iteration. In the following iteration, we need to evaluate the SNR when a new substream is assigned to this subcarrier. This computation requires the inversion of an augmented correlation matrix

$$\mathbf{R}(P+1) = \begin{bmatrix} \mathbf{R}(P) & \boldsymbol{\rho} \\ \boldsymbol{\rho}^H & 1 \end{bmatrix}, \quad (24)$$

where $\boldsymbol{\rho}=[\rho_{P+1,1}, \rho_{P+1,2\ldots}, \rho_{P+1,P}]^T$ is the vector of the cross-correlation coefficients between the new substream and P existing substreams. The inverse of $\mathbf{R}(P+1)$ can be computed by exploiting the existing $\mathbf{R}^{-1}(P)$ as follows [7]:

$$\mathbf{R}^{-1}(P+1) = \begin{bmatrix} \mathbf{R}^{-1}(P)+\mu\mathbf{R}^{-1}(P)\boldsymbol{\rho}\boldsymbol{\rho}^H\mathbf{R}^{-1}(P) & -\mu\mathbf{R}^{-1}(P)\boldsymbol{\rho} \\ -\mu\boldsymbol{\rho}^H\mathbf{R}^{-1}(P) & \mu \end{bmatrix} \quad (25)$$

where $\mu^{-1}=1-\boldsymbol{\rho}^H\mathbf{R}^{-1}(P)\boldsymbol{\rho}$. The number of multiplications and divisions required for this recursion is about $1.5(P+1)^2$, which is significantly smaller than that of the direct matrix inversion. The computational complexity of various allocation algorithms is summarized in TABLE I.

TABLE I. Computational complexity of allocation algorithms (for KN>>M case). In the proposed algorithm for the decorrelating detector, the recursion (24,25) is used.

Allocation algorithm	Complexity
WF	$\mathcal{O}(K^2N)$
Proposed algorithm for matched filter receiver	$\mathcal{O}(K^2N^2)$
Proposed algorithm for decorrelating detector	$\mathcal{O}(K^3N^3)$

V. NUMERICAL RESULTS

In the simulations, we assume that the subcarrier frequencies in (5) satisfy $f_{k,m}=f_{1,m}$ for all m, the delays in (8) are $\tau_k=\tau_0$ for all k, and the parameter $\beta=1$ in (12,14). Fig. 3 compares the average BER (over all users and fading realizations) of the WF algorithm, the optimal allocation algorithm and the proposed algorithm described in Section III for the matched filter receiver. Due to very high complexity of the optimal method, small number of substreams is employed in this comparison. We observe that the proposed algorithm incurs a small penalty for high E_b/N_0, while for low E_b/N_0, the proposed allocation algorithm outperforms the optimal method since the "optimality" criterion is not the average BER. As expected, WF algorithm has the worst performance among these three allocation algorithms in moderate to high E_b/N_0. Fig. 3 also demonstrates that the adaptive FH system with the proposed allocation algorithm provides 2-3dB power gain over the non-adaptive MC-CDMA with MRC in this system [2].

Fig. 4 compares the average BER of the WF and the proposed allocation algorithms for the matched filter receiver. We observe that the performance gain of the proposed algorithm over the WF method increases as the SNR grows. Note that we also investigated the modification of the WF algorithm suitable for the decorrelating detector. Significant loss was also observed relative to the proposed allocation method. This comparison corroborates our discussion on the limitations of WF algorithm in section III.

Fig. 5 illustrates the average BER of the proposed allocation algorithm for the decorrelating and matched filter detectors. The BER of the MF detector saturates due to high MAI, while the decorrelating detector has good performance. The single-user bounds given by the BER of MC-CDMA for one user that employs the same number of

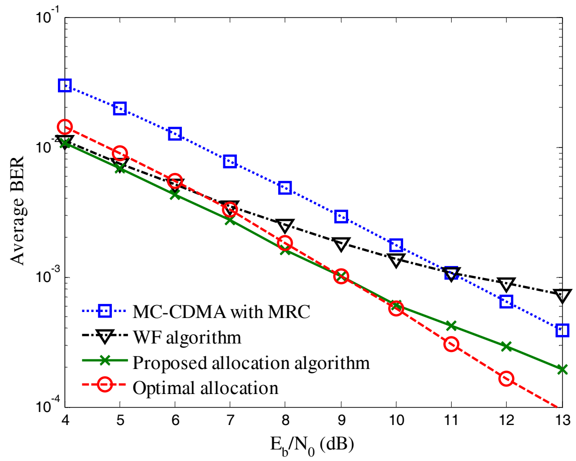

Fig. 3. BER performance of MC-CDMA system with adaptive FH/MRC for K=6 users, M=4 subcarriers, N=1 substream per user, and processing gain PG=64.

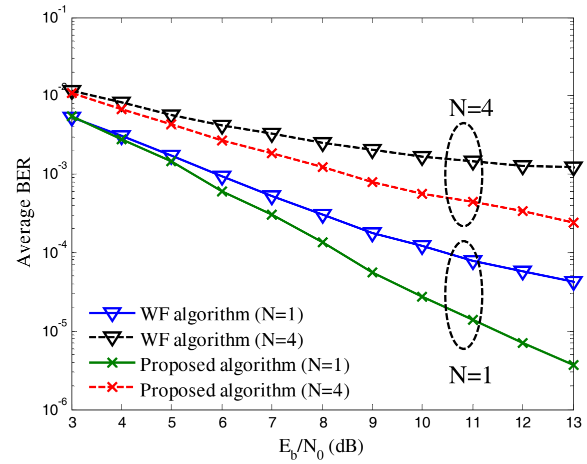

Fig. 4. BER performance of MC-CDMA system with adaptive FH for K=16, M=8, and PG=64.

subcarriers are also shown in Fig. 5. These single-user bounds are computed for MC-CDMA with MRC and MC-CDMA with adaptive FH. For the latter system, the allocation algorithm chooses the subcarrier that has the largest fading amplitude to transmit all substreams of the only user [3]. We observe that the average BER of the proposed algorithm for the decorrelating detector is very close to the single-user bound for the same system (MC-CDMA with adaptive FH). Thus, it is very effective in mitigating MAI in this system. Finally, we observe that this system achieves much lower BER than the single-user bound of MC-CDMA with MRC at the expense of increased complexity.

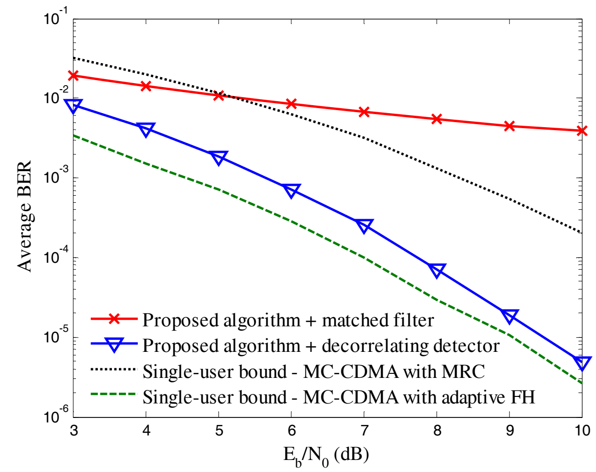

Fig. 5. BER performance of MC-CDMA system with adaptive FH for K=16, M=8, N=8, and PG=64.

VI. CONCLUSIONS

In this paper, we have proposed a novel allocation algorithm that outperforms the WF algorithm for MC-CDMA with adaptive FH for the reverse link. Moreover, we have demonstrated that the linear decorrelating detector that employs the proposed allocation algorithm is very effective in mitigating MAI, with performance approaching the single user bound for this system.

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