# Enabling Adaptive Modulation and Interference Mitigation for Slow Frequency Hopping Communications[1]

Ming Lei[+], Alexandra Duel-Hallen[+], Hans Hallen[*]

[+]North Carolina State University
Dept. of Electrical and Computer Engineering
Box 7911, Rayleigh, NC 27695-7911
Email: {mlei, sasha}@ncsu.edu

[*]North Carolina State University
Dept. of Physics
Box 8202, Rayleigh, NC 27695-8202
Email: hans_hallen@ncsu.edu

## ABSTRACT

We investigate the optimal Minimum Mean Square Error (MMSE) Long Range Prediction (LRP) algorithm for slow frequency hopping (SFH) systems that employ coherent detection. Statistical model of the prediction accuracy is developed and used in the design of reliable adaptive modulation techniques. Moreover, adaptive modulation is combined with adaptive transmitter frequency diversity to mitigate the effect of fading and partial-band interference in frequency hopping communications. Both standard Jakes model and a realistic non-stationary physical model are employed to test the performance. Analysis and simulation results show that significant performance gains can be achieved relative to non-adaptive methods.

## 1. INTRODUCTION

High speed wireless communications require efficient use of time-variant multipath fading channels. This creates a need for new transmission techniques that can adapt the transmission parameters to the channel variations. Adaptive transmission [1,2] improves spectral efficiency by transmitting the signal at high rate during favorable channel conditions, and reducing the rate as the channel conditions degrades. To achieve the potential of adaptive transmission, the channel state information (CSI) need to be known at the transmitter. The CSI is estimated by the receiver and fed back to the transmitter. Due to the delay associated with estimation and feedback, it is necessary to predict the channel several milliseconds ahead.

A novel long range prediction (LRP) algorithm for the flat fading channel was proposed in [3]. This algorithm benefits from using lower sampling rate than conventional techniques. This increases the memory span so that the channel can be predicted further into the future. In [4], LRP aided by observations at another carrier was investigated, and in [5], the LRP is extended to Orthogonal Frequency Division Multiplexing (OFDM) channels. A novel realistic physical model was developed to test this algorithm beyond the level of the standard Jakes model [6]. Using this model, performance of adaptive transmission aided by LRP was validated in [3, 4] for typical and challenging fading environments.

In this paper, we explore adaptive transmission aided by the LRP for slow frequency hopping (SFH) systems that employ coherent detection [9]. By exploiting the correlation between different frequencies, we propose to predict the channel coefficients in the next hopping interval of SFH systems based on a number of past fading observations from previous hopping intervals [7]. An adaptive transmission method for SFH systems was previously investigated in [10]. The goal was to improve the throughput efficiency by adapting to the slowly varying power of long-term fading and the interference level. In this paper, we adapt the modulation level and the transmission power to rapidly varying short-term channel variations using the LRP for FH channels. The objective is to further increase the spectral efficiency subject to the power and reliability constraints.

Partial-band Interference (PBI) is another serious source of degradation in FH communications. To mitigate the effects of PBI, coding can be utilized to decrease the BER [10]. In [11], a pre-whitening filter is used to reject interference in the fast frequency hopping (FFH) receiver. Diversity combining techniques have also been proposed for the FFH systems with non-coherent detection [12]. In this paper, we investigate joint adaptive frequency diversity and adaptive modulation to mitigate the effects of PBI and fading in SFH systems.

The remainder of this paper is organized as follows. In section 2, we describe the channel statistics and the MMSE LRP algorithm for SFH systems. Section 3 analyzes the performance of adaptive modulation aided by the LRP. This method is utilized jointly with adaptive frequency diversity combining for SFH channels with PBI in section 4.

[1] This research was supported by NSF grant CCR-0312294 and ARO grant DAAD 19-01-1-0638.

## 2. CHANNEL STATISTICS AND LONG RANGE PREDICTION FOR FREQUENCY HOPPING CHANNELS

Consider the SFH system that employs coherent detection with the total number of frequencies $q$ and the hopping rate $f_h$. Denote the frequency separation between adjacent frequencies as $\Delta f$. In this paper, we employ a randomly chosen periodic hopping pattern with length $N=q$, although the proposed methods are also applicable to non-periodic hopping patterns.

We use a Frequency Selective Gaussian Wide Sense Stationary Uncorrelated Scattering (GWSSUS) channel model for the FH channels [14]. Let $c(f(t),t)$ be the equivalent lowpass complex sample of the fading channel at time $t$ and frequency $f(t)$, where $f(t)$ is the carrier frequency occupied at time $t$. To simplify notation, we use $c(f,t)$ instead of $c(f(t),t)$. Assume fading is flat for each frequency. The spaced-time spaced-frequency correlation function with the time difference $\tau$ and the frequency separation $\Delta f$ is defined as $R(\Delta f,\tau)=\mathrm{E}[c(f,t)c^*(f+\Delta f,t+\tau)]$ [15].

From [14], the equivalent lowpass complex fading coefficients at frequency $f^i$ can be expressed as

$$c(f^i,t)=\sum_{n=1}^{N}A_n e^{j(2\pi f_n t+\varphi_{in})} \tag{1}$$

where $N$ is the number of reflectors. For the n$^{th}$ path, $A_n$ is the amplitude, and $f_n$ is the Doppler shift. The phase difference for the n$^{th}$ path between frequencies $f^i$ and $f^j$ is $\Delta\varphi_n = \varphi_{jn}- \varphi_{in} = -2\pi\Delta f T_n$ [14], where $\Delta f = f^{\,j}- f^{\,i}$ is the frequency separation, and $T_n$ is the excess delay of the n$^{th}$ path, which has exponential distribution [14].

The channel coefficient $c(f,t)$ is closely approximated by a zero mean complex gaussian random process. The time correlation function is given by $R_t(\tau) = J_0(2\pi f_{dm}\tau)$, where $J_0(\cdot)$ is a zero order Bessel function of the first kind and $f_{dm}$ is the maximum Doppler shift. The frequency correlation function $R(\Delta f) = \dfrac{1}{1+(2\pi\Delta f\sigma)^2}+j\,\dfrac{2\pi\Delta f\sigma}{1+(2\pi\Delta f\sigma)^2}$, where $\sigma$ is the rms delay spread [16]. Define $\Delta f\sigma$ to be the normalized frequency separation. From [13], $R(\Delta f,\tau)=R_t(\tau)R_f(\Delta f)$.

We employ the MMSE linear prediction method. Assume the channel coefficients in (1) are sampled at the rate $f_s=1/T_s$, and for an integer n, define $c(f(n),n)=c(f(nT_s),nT_s)$. The prediction $\hat{c}(f(n+\tau),n+\tau)$ ($\tau$ is a positive integer) of the future channel coefficient $c(f(n+\tau),n+\tau)$ based on $p$ past observations $c(f(n),n),\ldots, c(f(n-p+1),n-p+1)$ is formed as

$$\hat{c}(f(n+\tau),n+\tau)=\sum_{j=0}^{p-1}d_j(n)c(f(n-j),n-j) \tag{2}$$

The optimal prediction coefficients are computed as $\boldsymbol{d}(n)=\boldsymbol{R}(n)^{-1}\boldsymbol{r}(n)$, where $\boldsymbol{d}(n)=[d_0(n)\ldots d_{p-1}(n)]^T$, $\boldsymbol{R}(n)$ is the autocorrelation matrix with $R_{ij}(n)=\mathrm{E}\{c(f(n-i),n-i)c^*(f(n-j),n-j)\}$, and $\boldsymbol{r}(n)$ is the autocorrelation vector with $r_j(n)=\mathrm{E}\{c(f(n),n)c^*(f(n-j),n-j)\}$. The resulting instantaneous MMSE of this prediction method is $\mathrm{MMSE}(n)=1-\boldsymbol{d}(n)^{\mathrm{T}}\boldsymbol{r}(n)$ [15]. The MMSE for FH systems is computed as the average over all LP filters $\mathrm{MMSE}= 1/N\sum_{n=1}^{N}\mathrm{MMSE}(n)$. Because the hopping pattern is a random frequency sequence, a single prediction filter does not exist. The prediction coefficients, determined by the sampling time and the hopping pattern, need to be re-computed at the sampling rate.

When the channel correlation functions are not available at the transmitter, $R_t(\tau)$ and $R_f(\Delta f)$ must be estimated and updated when new observations are available. In our investigation, pilot symbol aided channel estimation is used to estimate the channel correlation functions [8].

The optimal MMSE LRP described above is complex, because it requires inversion of large matrices at the sampling rate. In [7], a recursive matrix update method was proposed. It significantly reduces the computation of the optimal LRP. Moreover, in [8], low complexity prediction methods were studied, and it was demonstrated that the optimal LRP method is required to achieve reliable prediction. In practice, $c(f(n),n)$ are observed in the presence of noise. The prediction coefficients $\boldsymbol{d}(n)$ can be easily modified to include the effect of the noise, and noise reduction methods can be utilized to reduce the noise present in the observations [3]. In this paper, we assume channel observations with SNR=100dB in (2).

In SFH systems, we predict the channel coefficients of the next dwell interval. A typical hopping rate of SFH systems is 500 hops/second. Thus, the prediction range $\tau T_s$=2ms is desirable. The sampling rate $f_s$=2kHz is employed due to its best performance for given parameters. Since the sampling rate is much lower than the symbol rate, interpolation is performed within a dwell interval to predict fading coefficients for all data points. As $p$ increases, the MMSE saturation level is approached. We found that the performance of LRP with $p=50$ has near-optimal performance [8]. Thus, we choose $p=50$ in our analysis throughout this paper.

## 3. ADAPTIVE TRANSMISSON AIDED BY LRP

We employ variable rate variable power Multiple Quadrature Amplitude Modulation (MQAM) [1,2]. First, assume fixed transmission power $E_s$ per symbol. The average Signal to Noise Ratio (SNR) is defined as $\bar{\gamma}=E_s/N_0$. Let $\alpha(t)=|c(f(t),t)|$ be the channel gain at time t. The instantaneous SNR $\gamma=\bar{\gamma}\alpha(t)^2$. Since $\hat{c}\,(f(n),n)$ in (2) is a linear combination of complex Gaussian random variables, it is also a zero mean complex Gaussian random variable. Thus, the predicted channel gain $\hat{\alpha}=|\hat{c}(f(n),n)|$ is

Rayleigh distributed. The conditional probability density function (pdf) of $\alpha$ given $\hat{\alpha}$ is [2]

$$p(\alpha|\hat{\alpha}) = \frac{2\alpha}{(1-\rho)\Omega} I_0\left(\frac{2\sqrt{\rho}\alpha\hat{\alpha}}{(1-\rho)\sqrt{\Omega\hat{\Omega}}}\right)\exp\left(-\frac{1}{(1-\rho)}\left(\frac{\alpha^2}{\Omega}+\frac{\rho\hat{\alpha}^2}{\hat{\Omega}}\right)\right) \quad (3)$$

where $\Omega$=E[$\alpha^2$] and $\hat{\Omega}$=E[$\hat{\alpha}^2$], and $I_0$ is the zero order modified Bessel function, $\rho=\mathrm{cov}(\alpha^2,\hat{\alpha}^2)/\sqrt{\mathrm{var}(\alpha^2)\mathrm{var}(\hat{\alpha}^2)}$ is the correlation coefficient between $\alpha^2$ and $\hat{\alpha}^2$. Define the adaptive modulation level selection method as follows: when $\alpha_i<\hat{\alpha}<\alpha_{i+1}$, $M(i)$-QAM is employed, where $M(1)=2$, $M(i)=2^{2(i-1)}$, $i=2,3,4$, $\alpha_5=\infty$. Given $\hat{\alpha}$, the BER of the selected modulation level is

$$\mathrm{BER}_{M(i)}^{*}(\overline{\gamma},\hat{\alpha}) = \int_0^{\infty} \mathrm{BER}_{M(i)}(\overline{\gamma}\alpha^2)\, p(\alpha|\hat{\alpha})\mathrm{d}\alpha \qquad (4)$$

where $\mathrm{BER}_{M(i)}$ is calculated using the upper bound of the BER of MQAM for AWGN channel [1]. The thresholds $\alpha_i$ are chosen as $\mathrm{BER}_{M(i)}^{*}(\overline{\gamma},\alpha_i)=\mathrm{BER}_{tg}$, where $\mathrm{BER}_{tg}$ is the target BER.

Once the modulation level $M$ is decided, an appropriate transmission power is found to maintain the target BER. We extend the discrete power control policy [1] to the case where the observations are predicted. In particular, once the thresholds are chosen as above, each modulation level is associated with a constant transmission power $E_s(i)$ selected to maintain the target BER.

Assuming the ideal Nyquist signal, the spectral efficiency takes on the same value as the average number of bits per symbol (BPS): $\hat{R}_{ada}=\sum_{i=1}^{4}\log_2 M(i)\int_{\alpha_i}^{\alpha_{i+1}} p_{\hat{\alpha}}(x)\mathrm{d}x$. For the discrete power control method, the average transmission power is $P_{avg}=\sum_{i=1}^{4}E_s(i)\int_{\alpha_i}^{\alpha_{i+1}} p_{\hat{\alpha}}(x)\mathrm{d}x$.

We first use the standard Jakes model to validate the performance of the proposed adaptive modulation method. In the evaluation, the spectral efficiency is computed numerically. The correlation coefficient $\rho$ is estimated by simulation. The maximum Doppler shift is $f_{dm}$=50Hz. The frequency hopping rate is 500Hz. A random hopping pattern with length $N=32$ is employed. The target BER is $10^{-3}$. The BPS of adaptive modulation as a function of normalized frequency separation $\Delta f\sigma$ is plotted in figure 1. We observe that the spectral efficiency degrades as $\Delta f\sigma$ increases. The SFH benefits from adaptive transmission primarily when $\Delta f\sigma$ does not significantly exceed 0.1. As $\Delta f\sigma$ grows, the spectral efficiency saturates and approaches that of non-adaptive modulation. Thus, for large $\Delta f\sigma$, the SFH will not benefit from adaptive transmission. However, the benefit of frequency diversity is greater as $\Delta f\sigma$ increases.

Typical values of the delay spread are on the order of microseconds in outdoor radio channels [16]. Suppose $\sigma$ is

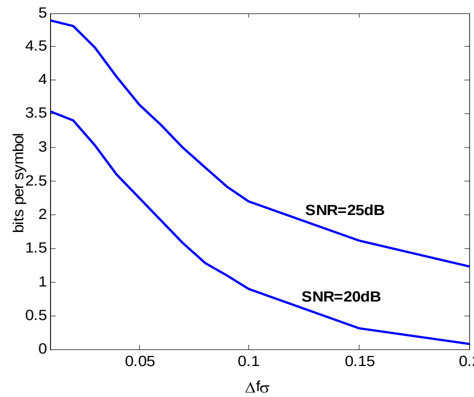


Fig.1. BPS of adaptive modulation vs. $\Delta f\sigma$.

1$\mu$s. A SFH system would benefit from adaptive transmission when the frequency separation is as large as 100 kHz. If $\sigma$ is 10$\mu$s, the frequency separation has to decrease to 10 KHz to obtain good performance. In realistic SFH systems [9], the symbol rate is on the order of tens of Ksps. Thus, the frequency separation of SFH systems is often less than 100 KHz. Therefore, adaptive transmission aided by the proposed channel prediction method is feasible for these systems. It was also demonstrated in [8] that adaptive transmission is beneficial for other typical SFH parameters and moderate to high Doppler shifts.

Next, we use our physical model to investigate the performance in realistic non-stationary fading channels. A typical scenario and a challenging scenario are created to test the performance. The reflectors are arranged to give an approximately exponential distribution of excess delays with the average $\sigma$=1$\mu$s for both scenarios. Figure 2 illustrates the variation of the rms delay spread $\sigma$ in these two scenarios. We observe that $\sigma$ changes slowly in the typical case, while in the challenging case, $\sigma$ changes rapidly over a wide range. The performance comparison is presented in Figure 3. Since the channel correlation functions vary faster in the challenging scenario than in the typical scenario, the prediction accuracy is worse in the challenging case. While the BPS gain is lower for the physical model than for the Jakes model due to the channel parameter variations, significant improvement is still obtained relative to non-adaptive modulation.

## 4. ADAPTIVE SFH WITH PARTIAL-BAND INTERFERENCE

Partial-band interference (PBI) is another source of degradation in the FH communications [15]. It is usually modeled as a narrow-band additive Gaussian noise, which occupies a small fraction $\delta$ of the bandwidth of the FH system. For the frequency slots where PBI is present, the power spectral density is $N_0+\delta^{-1}N_I$, where $N_I$ is the average power spectral density of the PBI. For the frequency slots without PBI, the power spectral density is $N_0$.

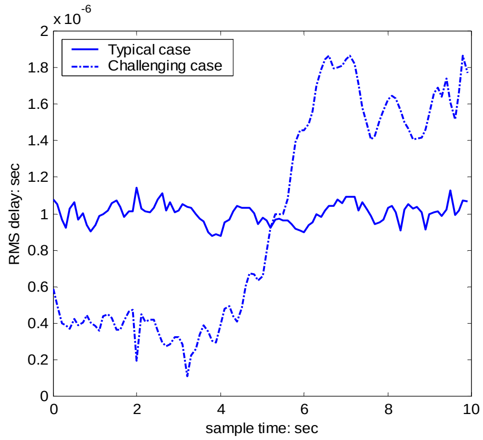


Figure 2. The variation of RMS delay σ.

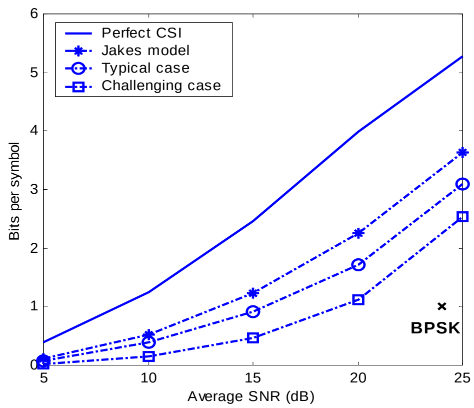


Figure 3. Performance comparison for the Jakes model and the physical model.

We propose to use adaptive frequency diversity (referred to as diversity FH) to mitigate the effects of PBI. To reduce the complexity and to simplify the analysis, just 2 frequencies are chosen. Each symbol is transmitted at two frequencies $f^1$ and $f^2$ simultaneously according to a hopping pattern. These two hopping frequencies are chosen to have large frequency separation to reduce the frequency correlation. Thus, we can assume independent fading at the two hopping frequencies.

We assume that the receiver knows perfectly where the PBI is present. The transmitter monitors the two upcoming frequencies $f^1$ and $f^2$ prior to the transmission [17]. Let $I_k$ denote the indicator function for the presence of PBI at the upcoming frequency $f^k$, $I_k=1$ if PBI is present at $f^k$, and $I_k=0$ if PBI is absent at $f^k$. Since the time-variant nature of the interference induce uncertainty into the knowledge of the PBI, we introduce a reliability factor η to model the reliability of the knowledge of the PBI at the transmitter. The probability of the PBI at frequency $f^k$ is calculated as $p_k=\eta I_k+(1-\eta)(1-I_k)$, where $\eta\in[0,1]$. As η increases, the reliability improves.

The receiver employs diversity combining techniques [15] to form the decision statistics. When there is no PBI at both $f^1$ and $f^2$, Maximal Ratio Combining (MRC) is used. When one frequency has PBI, the receiver chooses the frequency that does not has PBI. If both frequencies have PBI, we assume an error occurs. The proposed adaptive modulation method operates as follows. Let $f^1$ and $f^2$ be the two upcoming frequencies, and $\hat{\alpha}_1$ and $\hat{\alpha}_2$ be the predicted channel gains, respectively. The average BER when $M(i)$-QAM is employed by the transmitter is

$$\mathrm{BER}^*_{M(i)}(E_s,N_0,\hat{\alpha}_1,\hat{\alpha}_2,p_1,p_2)=\frac{(1-p_1)(1-p_2)}{1-p_1p_2}\times$$
$$\int_0^\infty\int_0^\infty \mathrm{BER}_{M(i)}\Big(\gamma=\frac{E_s(\alpha_1^2+\alpha_2^2)}{N_0}\Big)p(\alpha_1|\hat{\alpha}_1)p(\alpha_2|\hat{\alpha}_2)d\alpha_1 d\alpha_2$$
$$+\frac{p_2(1-p_1)}{1-p_1p_2}\int_0^\infty \mathrm{BER}_{M(i)}\Big(\gamma=\frac{E_s\alpha_1^2}{N_0}\Big)p(\alpha_1|\hat{\alpha}_1)d\alpha_1$$
$$+\frac{p_1(1-p_2)}{1-p_1p_2}\int_0^\infty \mathrm{BER}_{M(i)}\Big(\gamma=\frac{E_s\alpha_2^2}{N_0}\Big)p(\alpha_2|\hat{\alpha}_2)d\alpha_2+0.5p_1p_2. \quad (5)$$

The modulation level is chosen as

$$\tilde{M}=\max\{M(i)\mid \mathrm{BER}^*_{M(i)}(E_s,N_0,\hat{\alpha}_1,\hat{\alpha}_2,p_1,p_2)\le \mathrm{BER}_{tg}\} \quad (6)$$

In the LRP for channels with PBI, past observations at interference-free dwell intervals are used. The optimal MMSE LRP algorithm with recursive autocorrelation matrix update is utilized. While PBI degrades the accuracy of the LRP, the quality of prediction is improved relative to interference-free system without diversity [8].

Next, we describe a sentient FH diversity technique that can further improve the performance of SFH systems. In this method, channel coefficients of $L$ widely spaced frequencies are predicted, and a subset of $r$ frequencies with the largest channel gains are selected in the transmission. In the presence of PBI, the average BER for each modulation level is computed, and the largest signal constellation that satisfies the BER requirement is selected for the transmission.

Simulations are used to demonstrate the performance of adaptive SFH with PBI for the Jakes model with the same parameters as in section 3. In figure 4, we plot the BPS of adaptive modulation using diversity FH and sentient FH under the assumption of perfect knowledge of PBI at the transmitter. As expected, the performance degrades as δ increases. We observe that the performance of sentient FH is better than that of diversity FH due to its larger channel gain. While performance of sentient FH improves with increasing $L$ when $L$ is moderate, as $L$ becomes very large, the BER actually increases due to inaccurate frequency selection. Our investigation shows that $L=4$ has near-optimal performance [8]. We also plot the BPS of adaptive modulation for a non-diversity system with PBI. In this case, adaptive modulation as in (4) is applied when the upcoming frequency is interference-free, and outage occurs if there is interference. Note that when this method is extended to $\eta<1$, the target BER cannot be satisfied, implying that diversity is required for channels with imperfect knowledge of PBI at the transmitter. Figure 5 shows the BPS of adaptive diversity FH systems as a function of η. We observe that the spectral efficiency degrades as η decreases. For both diversity and sentient FH, when η≤*0.95*, the target BER cannot be satisfied with the uncoded adaptive transmission method proposed above, and coding methods that provide further diversity are required. However, this investigation is beyond the scope of this paper.

## 5. CONCLUSION

The optimal MMSE long range channel prediction algorithm for SFH communications with coherent detection was introduced, and was shown to enable adaptive modulation for SFH. Joint adaptive diversity combining and adaptive modulation were investigated for SFH communications with PBI and shown to improve performance significantly relative to non-adaptive methods.

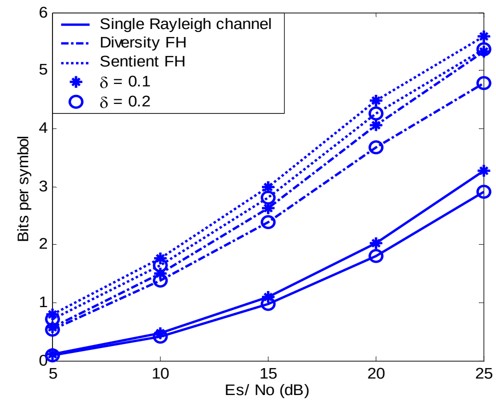

Fig. 4. Performance of adaptive SFH with PBI

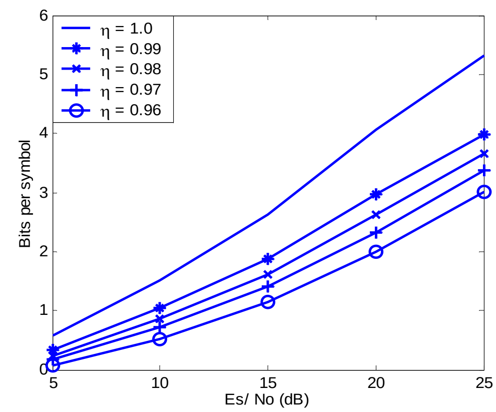

Fig. 5. Performance of adaptive SFH with PBI

## 6. REFERENCES

[1] A.J. Goldsmith and S.G. Chua, "Variable-Rate Variable-Power MQAM for fading channels", IEEE Trans Comm, vol. 45, No. 10, Oct. 1997, pp. 1218-1230.
[2] D.L. Goeckel, "Adaptive Coding for Time-Varying Channels using Outdated Channel Estimation", IEEE Trans. on Communications, vol.47, No.6, June 1999.
[3] A. Duel-Hallen, S. Hu, H. Hallen, "Long Range Prediction of Fading Signal: Enabling Adaptive Transmission for Mobile Radio Channels", IEEE Signal Processing Mag., vol.17, No.3, pp. 62-75, May 2000.
[4] T.S. Yang, A. Duel-Hallen, H. Hallen, "Reliable Adaptive Modulation Aided by Observations of Another Fading Channel", IEEE Trans. on Comm., Vol. 52, NO. 4, Apr. 2004, pp. 605-611.
[5] T.S. Yang, A. Duel-Hallen, H. Hallen, "Enabling Adaptive OFDM for Mobile Radio Channels", Proc. of MILCOM'04.
[6] H. Hallen, A. Duel-Hallen, S. Hu, "A Physical Model for Wireless Channels to Provide Insight for Long Range Prediction", Proc. of MILCOM'02, Oct. 7-10, 2002.
[7] M. Lei, A. Duel-Hallen, "Long Range Channel Prediction and Adaptive Transmission For Frequency Hopping Communications", Proc. of 41th Annual Allerton conference on Comm., Control, and Computing, Oct. 1-3, 2003, , pp. 1476-1485.
[8] M. Lei, "Performance Analysis of Reliable Adaptive Transmission for Mobile Radio Slow Frequency Hopping Channels Aided by Long Range Prediction", Ph.D dissertation, NC State University, Fall 2004.
[9] S. Tomisato, K. Fukawa, "Coherent Frequency Hopping Multiple Access (CFHMA) with Multiuser Detection for Mobile Communication Systems", IEEE Trans. Vehi. Tech., Vol. 49 Issue: 2, March 2000, pp. 531–539.
[10] M.B. Pursley, C.S. Wilkins, "Adaptive-Rate Coding for Frequency-Hop Communication over Rayleigh Fading Channels", IEEE Journal on Selected Area in Communications, vol.17, No.7, July 1999, pp. 1224-1232.
[11] R.A. Iltis, J.A. Ritcey, "Interference rejection in FFH systems using least square estimation techniques", IEEE Trans. on Comm., vol. 38, issue 12, Dec. 1990, pp.2174-2183.
[12] C.M. Keller, M.B. Pursley, "Diversity Combining for Channels with Fading and Partial-Band Interference", IEEE JSA C, Vol. SAC-5, No. 2, Feb. 1987. pp. 248-260.
[13] Y. Li, L. J. Cimini, "Robust Channel Estimation for OFDM Systems with Rapid Dispersive Fading Channel," IEEE Trans. Commun., vol. 46, pp. 902-915, Apr. 1998.
[14] W.C. Jakes, Microwave Mobile Communications. Wiley, New York, 1974.
[15] J.G. Proakis, Digital Communications. 4th Edition, McGraw Hill, 1996.
[16] T.S. Rappaport, Wireless Communications: Principles and Practice. Prentice-Hall, 1996.
[17] O.C. Yue, R. Yeung, R.S. Bateman, S. Clark, "Performance of a frequency hopping CSMA packet radio network", Proc. of IEEE MILCOM 1990, vol.1, pp: 58 – 64.