

# Adaptive Modulation Using Long Range Prediction for Flat Rayleigh Fading Channels<sup>1</sup>

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**Abstract ---** We theoretically analyze the statistical behavior of prediction errors generated by our previously proposed long range prediction algorithm, and investigate adaptive modulation design using predicted channel state information (CSI). Both numerical and simulation results show that accurate prediction of the fading channel far ahead makes adaptive transmission feasible for rapidly time-varying mobile radio channels.

## 1. Introduction

Adaptive modulation methods depend on accurate channel state information (CSI) that can be estimated at the receiver and sent to the transmitter via a feedback channel. This information would allow the transmitter to choose the appropriate transmitted signal. The feedback delay and overhead, processing delay and practical constraints on modulation switching rates have to be taken into account in the performance analysis of adaptive modulation methods. For very slowly fading channels (pedestrian or low vehicle speeds), *outdated* CSI is sufficient for reliable adaptive system design. However, for rapidly time variant fading that corresponds to realistic mobile speeds, even small delay will cause significant degradation of performance since channel variation due to large Doppler shifts usually results in a different channel at the time of transmission than at the time of channel estimation [1, 2]. To realize the potential of adaptive transmission methods, these channel variations have to be *reliably predicted* at least several milliseconds ahead.

Recently, we have investigated a *novel adaptive long-range fading channel prediction algorithm* in [3]. This algorithm characterizes the fading channel using an autoregressive (AR) model and computes the Minimum Mean Squared Error (MMSE) estimate of a future fading coefficient sample based on a number of past observations. The superior performance of this algorithm relative to conventional methods is due to its *low sampling rate* [3]. Given a *fixed model order*, the lower sampling rate results in *longer memory span*, permitting prediction further into the future. The prediction method is enhanced by an *adaptive tracking* method [3] that increases accuracy, reduces the effect of noise and maintains the robustness of long-range prediction as the physical channel parameters vary.

In this paper, we extend the application of long range channel prediction to adaptive modulation. First, we theoretically analyze the statistical behavior of prediction errors generated by our long range prediction algorithm, and consider adaptive modulation design based on this prediction error model using predicted CSI. Then, we evaluate the performance of adaptive modulation for flat Rayleigh fading channels. The extension of this method to our novel realistic non-stationary fading model and measured data are discussed in [3,4] and references therein.

## 2. Results

Consider the linear MMSE prediction of the future channel sample  $\hat{c}_n$  based on  $p$  previous samples  $c_{n-1}, \dots, c_{n-p}$  as [3]:

$$\hat{c}_n = \sum_{j=1}^p d_j c_{n-j} \quad (1)$$

where the coefficients  $d_j$  are determined by the orthogonality principle. We assume that channel samples  $c_n$  are modeled as zero-mean complex Gaussian random variables, i.e., the channel is Rayleigh fading. Thus, the amplitude  $\alpha = |c_n|$  and its predicted value  $\hat{\alpha} = |\hat{c}_n|$  have a bivariate Rayleigh distribution. We define the prediction error  $\beta$  as the ratio of the actual fading gain  $\alpha$  and the predicted fading gain  $\hat{\alpha}$ , i.e.,  $\beta = \alpha/\hat{\alpha}$ . Then the probability density function (pdf) of  $\beta$  can be derived as:

$$p_\beta(x) = \frac{2x(\sqrt{\lambda}x^2 + \lambda)(1-\rho)}{((\sqrt{\lambda}x^2 + \lambda)^2 - 4\rho x^2)^{1.5}}, \quad (2)$$

where the correlation coefficient  $\rho = \frac{\text{Cov}(\alpha^2, \hat{\alpha}^2)}{\sqrt{\text{Var}(\alpha^2)\text{Var}(\hat{\alpha}^2)}}$ ,  $0 < \rho < 1$ ,  $\Omega = E\{\alpha^2\}$ ,  $\hat{\Omega} = E\{\hat{\alpha}^2\}$ , and  $\lambda = \sqrt{\Omega/\hat{\Omega}}$ .

We consider the fixed power and modulation level-controlled scheme using Square Multilevel Quadrature Amplitude Modulation (MQAM) signal constellation for the target Bit Error Rate (BER<sub>t</sub>) = 10<sup>-3</sup>. We restrict ourselves to MQAM constellations of sizes  $M = 0, 2, 4, 16, 64$ . Given fixed transmitter power  $E_s$  (or the average Signal-to-Noise Ratio (SNR) level  $\bar{\gamma} = E_s/N_0$ ), to maintain a target BER, we need to adjust the modulation size  $M$  according to the instantaneous channel gain  $\alpha(t)$ . In other words, the adaptive modulation scheme can be specified by the threshold values  $\alpha_i$ ,  $i = 1, \dots, 4$ , defined as: when  $\alpha(t) \geq \alpha_i$ ,  $M_i$ -QAM is employed, where  $M_1 = 2$ ,  $M_i = 2^{2(i-1)}$ ,  $i > 1$ . When perfect CSI  $\alpha(t)$  is available, these thresholds can be directly calculated from the BER bound of MQAM for an Additive White Gaussian Noise (AWGN) channel [1]:

$$\text{BER}_M \leq 0.2 \exp(-1.5\gamma(t)/(M-1)) \text{ for } M > 2, \text{ and} \\ \text{BER}_2 = Q(\sqrt{2\gamma}) \quad (3)$$

where  $\gamma(t) = \alpha^2(t)\bar{\gamma}$  is the instantaneous received SNR. However, when the predicted CSI  $\hat{\alpha}(t)$  is used, the current channel condition is characterized by the distribution of  $p(\alpha/\hat{\alpha})$  which can be calculated as:

$$p_{\alpha/\hat{\alpha}}(x) = \frac{1}{\hat{\alpha}} p_\beta\left(\frac{x}{\hat{\alpha}}\right) \quad (4)$$

Then, the BER bound for predicted CSI  $\hat{\alpha}$ , say  $\text{BER}_M^*$ , can be obtained by evaluating the expectation of  $\text{BER}_M$  over  $\beta$  using  $p_\beta(x)$  in (2) as:

$$\text{BER}_M^* = \int_0^\infty \text{BER}_M(\sqrt{\lambda}x^2\hat{\alpha}^2) p_\beta(x) dx \quad (5)$$

This indicates that we need to use  $\text{BER}_M^*$  rather than  $\text{BER}_M$  to calculate thresholds when only the predicted CSI is available. In our study, we found that when our long range prediction is used for the realistic prediction range, there is small difference between the thresholds calculated using perfect CSI and predicted CSI [4]. This demonstrates that the long range prediction preserves the ideal bit rate while maintaining the target BER. However, from the results in [2], we found that even very small delay will cause great loss of bit rate for fast vehicle speeds when the strongly robust signaling design rule is used without long range prediction. Thus, *accurate long-range prediction is required to achieve the bit rate gain of adaptive MQAM for rapid vehicle speeds and realistic delays.*

## References

- [1] A. J. Goldsmith and S.G. Chua, "Variable-Rate Variable-power MQAM for Fading Channels", *IEEE Trans Comm*, vol. 45, No 10, pp 1218-1230, Oct 1997.
- [2] D. L. Goeckel, "Adaptive Coding for Fading Channels using Outdated Channel Estimates", *Proceedings of VTC*, May 1998.
- [3] A. Duel-Hallen, S. Hu, H. Hallen, "Long-range Prediction of Fading Signals: Enabling Adaptive Transmission for Mobile Radio Channels", to appear in *Signal Processing Magazine*, May 2000.
- [4] S. Hu, A. Duel-Hallen, H. Hallen, "Long-Range Prediction Makes Adaptive Modulation Feasible for Realistic Mobile Radio Channels," *Proc. of 34rd Annual Conf. on Infor. Sciences and Systems*, March 2000, Vol I, pp.WP4-7-WP4-13.

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