Improved Viterbi Decoder Metrics for Two-Stage Detectors in DS-CDMA

Ayman Elezabi, Alexandra Duel-Hallen, *Member*

***Abstract*— Modified branch metrics are proposed for single-user Viterbi decoders in two-stage detectors for convolutionally-encoded code-division multiple-access (CDMA) systems with random spreading sequences. The modifications are based on modelling the residual multiple-access interference (RMAI) after subtractive interference cancellation as conditionally Gaussian with time-dependent variance, where the conditioning is on the time-varying user crosscorrelations. A novel estimate of the variance of the total RMAI is presented, and used in the proposed branch metrics. Significant performance gains are demonstrated over the Euclidean branch metric of the standard Viterbi decoder.**

***Index Terms*— CDMA, subtractive interference cancellation, single-user Viterbi decoders, modified branch metrics, two-stage detectors, residual multiple-access interference, time-dependent variance.**

I. INTRODUCTION

VIRTUALLY all multiple-access communication systems today employ some form of forward error correction. Code-Division Multiple-Access (CDMA) systems may, in addition, employ multiuser detection to improve performance beyond that possible with single-user detectors. For asynchronous convolutionally encoded systems, the maximum-likelihood (ML) joint multiuser detector and decoder [1], however, has complexity that is exponential in the product of the number of users and the constraint length of the encoder. On the other hand, complete partitioning of the multiuser detection and decoding functionality in the receiver may limit performance considerably. One practical solution to the problem, therefore, is to pass information between the multiuser detector and a bank of *single-user* decoders. Examples of this approach include [2] where the multi-user detector produces reliability information to be used by a bank of outer single-user decoders. More recent schemes, e.g. [3], use reliability-dependent IC and several iterations between single-user maximum-a-posteriori (MAP) decoders and the multiuser detector.

Considerably less complex structures using single-user Viterbi decoders producing hard outputs were proposed in [4] for a code-spread CDMA system using successive IC and in [5], among other proposals, for a conventional CDMA system with parallel IC. Subtractive IC receivers, also called multistage detectors [6], naturally possess the potential for utilizing the single-user decoders to improve the IC operation itself. In all the receiver structures employing Viterbi decoders (VDs) the standard Viterbi decoder (VD) branch metric, which is based on the AWGN model, was used. Yet the structured multiple-access interference (MAI), or residual MAI (RMAI) after IC, in a multiuser system has statistics that are quite different from that of Gaussian interference. Potential improvements in the single-user decoders have, therefore, been largely overlooked. In this paper, we propose improved branch metrics for the single-user VDs following a two-stage detector [6] with the conventional first stage for a CDMA system using long spreading sequences, such as IS-95 [7]. The idea is based on modelling the RMAI as Gaussian with time-dependent variance after conditioning on the time-varying user crosscorrelations [8]. A novel estimate of the variance of the RMAI is derived for that purpose. We consider two alternative receiver structures, where in the first one decoding is performed only after the IC operation and in the second structure decoding is performed after each stage [9]. Modified VD metrics are proposed for both structures and we demonstrate performance improvements for both although the gains are more significant for detectors where IC is carried out completely before decoding.

The paper is organized as follows. Section 2 gives a description of the CDMA system model that is used throughout the paper. In section 3, the modified VD metrics are motivated and derived for the two alternative receiver structures and performance comparisons are given. A discussion of results and conclusions are presented in section 4.

II. CDMA SYSTEM MODEL AND RECEIVER STRUCTURES

Consider K users transmitting synchronously using binary CDMA signaling over a flat Rayleigh fading channel. At the receiver, a bank of K matched filter correlators despreads each user's signal. Sampling at the bit rate, we can write the output of the correlator bank for a given sample point at baseband as

$$y_k(1) = c_k b_k + \sum_{j=1, j\neq k}^{K} r_{kj} c_j b_j + n_k \qquad k = 1, \ldots, K \quad (1)$$

where the argument in the parentheses denotes the stage number, $c_k = |c_k| e^{\jmath\theta_k}$ are independent zero-mean complex Gaussian fading coefficients, $b_k \in \{-1, +1\}$ is the data bit of user k, n_k is a zero-mean complex Gaussian additive noise term with variance $\sigma^2 = \frac{1}{2}\mathbf{E}[n_k^* \, n_k]$, and r_{kj} is the normalized crosscorrelation between users k and j. The covariance between the real as well as between the imaginary parts of

Manuscript received June 10, 2003; revised November 18, 2003.

A. Elezabi is with the American University in Cairo and A. Duel-Hallen is with North Carolina State University.

This research was supported by NSF grants CCR-9725271 and CCR-981-5002 and ARO grant DAA-19-01-1-0638

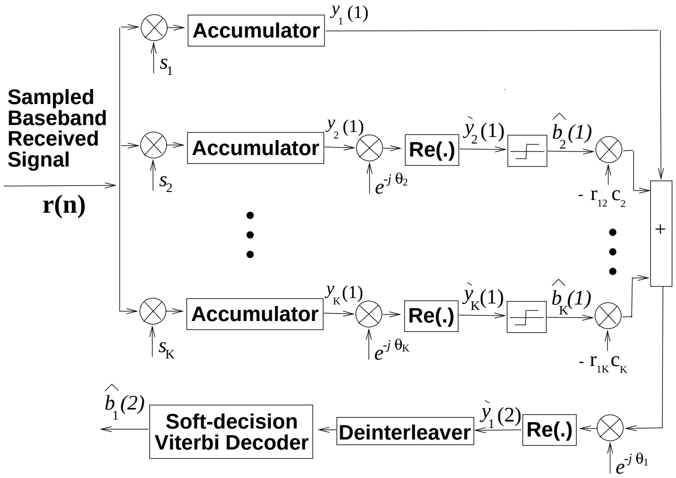

Fig. 1. Conceptual Diagram of ICUD (showing IC for user 1 only)

n_j and n_k is equal to $r_{kj}\sigma^2$, whereas the real and imaginary components of any noise term are independent. The signal-to-noise ratio (SNR) for user j is defined as $\gamma_j = \frac{1}{2}\frac{\mathbf{E}[|c_j|^2]}{\sigma^2}$. For coherent reception, the single-user, or conventional, first stage decision about the bit of user k is given by $\hat{b}_k(1) = \text{sgn}[\grave{y}_k(1)]$ where

$$\begin{aligned}\grave{y}_k(1) &= \mathbf{Re}[e^{-\jmath\theta_k} y_k(1)] \\ &= |c_k| b_k + \sum_{j=1, j\neq k}^{K} r_{kj}|c_j|\beta_{jk} b_j + \grave{n}_k \end{aligned} \quad (2)$$

where $\beta_{jk} = \cos(\theta_j - \theta_k)$, and $\{\grave{n}_j\}$ have the same joint distribution as $\{\mathbf{Re}(n_j)\}$ or $\{\mathbf{Im}(n_j)\}$. The output of the second stage, i.e. after one stage of IC, for user 1 (henceforth, our user of interest) is given by

$$\begin{aligned}\grave{y}_1(2) &= |c_1| b_1 + \sum_{j=2}^{K} 2r_{1j}|c_j|\beta_{j1} e_j + \grave{n}_1 \\ &\triangleq |c_1| b_1 + \zeta + \grave{n}_1 \end{aligned} \quad (3)$$

where $e_j = \frac{1}{2}(b_j - \hat{b}_j(1))$ represents the error in the first stage decision of user j and $\zeta \triangleq \sum_j \zeta_j$ is the total residual MAI (RMAI), where ζ_j is the RMAI due to user j. For the uncoded system, the final decisions are given by $\hat{b}_k(2) = \text{sgn}[\grave{y}_k(2)]$.

Now, suppose each user's data is convolutionally encoded and interleaved, where b_j now refers to the code bit of user j during the interval of interest. We consider the interleaver size to be sufficient to render the fading coefficients uncorrelated from one bit interval, or sample point, to another. Throughout the paper we use the half-rate convolutional encoder given by the octal generators 5 and 7 and that has a memory order equal to 2. The decoding may be performed only once after the IC stage(s), or it may be performed after each stage. We first consider modified VD metrics for the former structure, which we refer to as Interference Cancellation with Undecoded Decisions (ICUD). A conceptual diagram of this scheme is shown in Fig. 1. This represents a complete partitioning of multiuser detection and decoding. The second structure we consider is the post-decoding IC (PDIC) receiver where a bank of single-user VDs prior to the second stage results in better tentative decisions on the *code* bits $\{\hat{b}_j(1)\}$. A second bank of VDs is of course needed after the IC operation. A conceptual block diagram of this scheme is shown in Fig. 2. The interleavers in the receiver are required to re-order the MAI estimates for IC. Except in a few special cases [9], the PDIC scheme is superior to the ICUD in performance when distinct interleaving patterns (DIP) are assigned to the users, but it is generally more complex. The DIP scheme is needed to randomize the error bursts coming out of the first VD bank, which are detrimental to the performance of the second VD bank [9].

From the above description, we note that subtractive IC detectors generally require the channel estimates, crosscorrelation values (or spreading sequences), and also timing estimates of all interfering users. The single-user decoders may also benefit from such information, particularly since it is available or acquired at the IC stages. In the following section, we propose improved VD branch metrics for both ICUD and PDIC receiver structures based on this idea.

III. MODIFIED VITERBI DECODER BRANCH METRICS

The modified VD branch metrics are based on how we model the RMAI in the signal of our user of interest. Consider the RMAI terms $\zeta_j = 2r_{1j}|c_j|\beta_{j1}e_j$. For deterministic sequences, the sum of these terms plus the additive Gaussian noise was found to have a probability density function that can be reasonably approximated by the Gaussian density. This is supported by application of the Central Limit Theorem (CLT) for dependent variables[1] [10], as well as the fact that the factor $|c_j|\beta_{j1}$ in ζ_j is Gaussian. Note that e_j takes the values $0, 1,$ or -1. Yet even when conditioning on $e_j \neq 0$, ζ_j itself is not exactly Gaussian due to the dependence between e_j and $|c_j|\beta_{j1}$. For systems with long (random) sequences, on the other hand, the RMAI terms $\{\zeta_j\}$ involve additional randomness due to the time-varying crosscorrelations $\{r_{1j}\}$. The total RMAI is "less Gaussian" in this case, which is detrimental to the performance of the VD bank since the standard branch metric used in the VD is based on the assumption of Gaussian interference. Now consider ζ_j as the product of the approximately Gaussian random variable $(|c_j|\beta_{j1}e_j)$ and r_{1j}. The crosscorrelations can be calculated at the receiver. Thus, conditioning on $\{r_{1j}\}$, the total RMAI ζ plus noise may be modelled as a Gaussian random variable with time-dependent variance. The variance is time-dependent due to the changes in the values of the crosscorrelations $\{r_{1j}\}$ from one code bit interval to the next. The ML VD metric for AWGN channels with time-dependent variance can be easily derived from basic principles and has appeared in, for example, [11]. Referring to user 1, given the channel coefficients $\{|c_{1,i}|\}$ and crosscorrelations $\{r_{1j,i}\}$, where i denotes the $i-$th code bit interval, and due to the memoryless nature of the channel, the ML bit sequence is that which maximizes the conditional

[1]It is stated in [10] that the conditions for applicability of the CLT to a sum of dependent random variables are relatively mild, but difficult to verify. Therefore, we rely on agreement between our model and computer simulations to satisfy this claim.

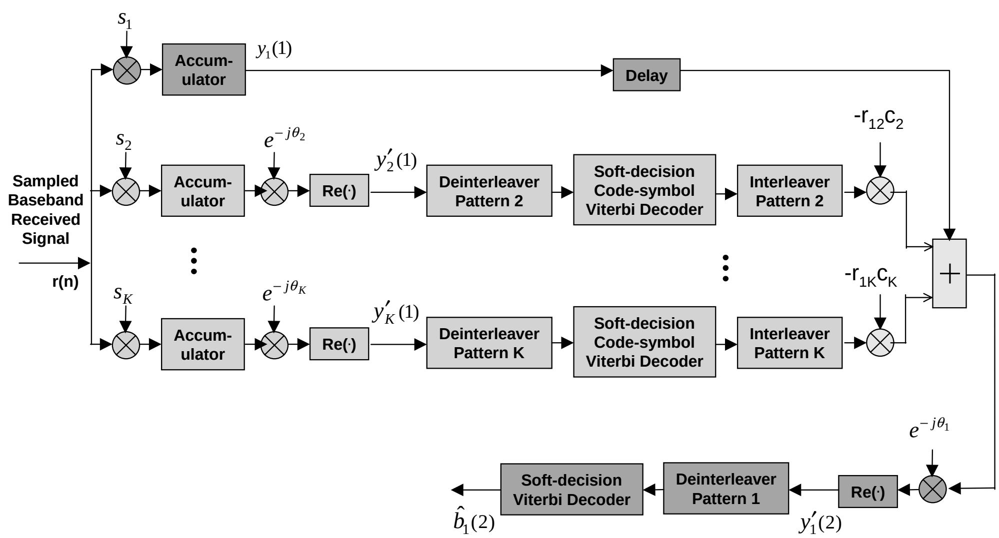

Fig. 2. Conceptual Diagram of PDIC (showing IC for user 1 only)

probability density function

$$f(\grave{y}_{1,i}(2)|\{|c_{1,i}|, b_{1,i}, r_{1j,i}\}) = \prod_i \frac{1}{\sqrt{2\pi\psi_{1,i}}} e^{-[\grave{y}_{1,i}(2)-|c_{1,i}|b_{1,i}]^2/2\psi_{1,i}} \quad (4)$$

where $\psi_{1,i} = \mathbf{Var}[\zeta + \grave{n}_1|\{r_{1j,i}\}] \approx \mathbf{Var}[\zeta|\{r_{1j,i}\}] + \sigma^2$ and $\grave{y}_{1,i}(2)$ is the soft output after IC as given in (3) with the added subscript i denoting time. Taking the logarithm, and dropping terms irrelevant to the maximization, the branch metric that is used in the VD to recursively maximize the above density becomes

$$[\grave{y}_{1,i}(2) - |c_{1,i}|b_{1,i}]^2/\psi_{1,i} \quad (5)$$

The task now becomes that of finding the conditional variance $\psi_{1,i}$.

A. Improved VD Metrics for ICUD structures

We prove in the appendix that $\psi_{1,i}$, the variance of the total RMAI plus noise seen by user 1 conditioned on the crosscorrelations $\{r_{1j,i}\}$, for systems with random spreading sequences may be approximated by

$$\psi_{1,i} \approx 2\sum_{j=2}^{K} r_{1j,i}^2 \mathcal{E}_j \left[1 - \sqrt{\frac{\mathcal{E}_j}{\mathcal{E}_j + \eta_j^2}} \left(\frac{3\eta_j^2 + 2\mathcal{E}_j}{2\eta_j^2 + 2\mathcal{E}_j}\right)\right] + \sigma^2 \quad (6)$$

where

$$\eta_j^2 = \frac{1}{N}\sum_{l \neq j} \mathcal{E}_l + \sigma^2 \quad (7)$$

represents the unconditional variance of the MAI (i.e. at the first stage) plus noise seen by user j, $\mathcal{E}_j = \frac{1}{2}\mathbf{E}[|c_j|^2]$ is the average code bit energy for user j, and N is the spreading factor which is equal to the number of chips per code bit. Note that η_j^2 is not exactly equal to the true unconditional variance since we already conditioned on r_{1j} but the difference is negligible. We denote the modified branch metric thus obtained by M1.

A more accurate estimate of $\psi_{1,i}$ may be obtained if we condition on $\{r_{lj,i} : l, j = 1, \ldots, K \text{ and } l \neq j\}$, i.e. on all the users' crosscorrelations rather than just the crosscorrelations between user 1 and the other users. In that case, we replace η_j^2 in (6), and in its derivation given in the appendix, by

$$\eta_{j,i}^2 = \sum_{l \neq j} \mathcal{E}_l \; r_{jl,i}^2 + \sigma^2 \quad (8)$$

This is the MAI variance conditioned on all user crosscorrelations plus noise seen by user j. We denote the metric based on using (8) in (6) by M2. The main difference between metrics M1 and M2 is that for M1 we use the average of the square of the user crosscorrelations, $\mathbf{E}[r_{lj}^2] = 1/N$, to obtain the unconditional MAI variance η_j^2 whereas for M2 we condition on all user crosscorrelations to obtain the conditional MAI variance $\eta_{j,i}^2$. The metric M1 is of course simpler to compute than the metric M2.

Fig. 3 shows the performance improvement due to the modified soft-decision VD metrics M1 and M2 for 8 users with $\grave{N} = 32$ on a frequency-nonselective Rayleigh fading channel. $\grave{N} = 2N$ for half-rate codes and is the number of chips per information bit. Performance comparisons are given in terms of $\grave{N}$ since it represents the overall bandwidth expansion due to the spreading sequences and the encoding. Likewise, in all BER comparisons we shall plot the BER against the

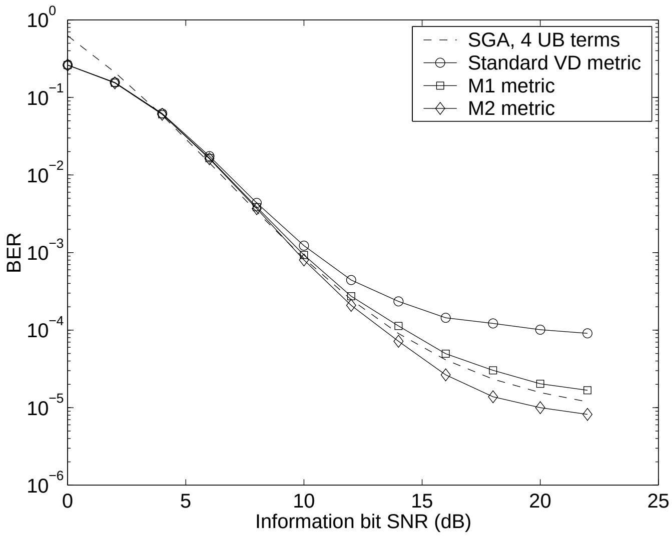

Fig. 3. Comparison of modified and standard VD metrics for 8 users with $\grave{N} = 32$ using a half-rate, memory order 2, code and soft-decision decoding with the ICUD scheme on a flat Rayleigh fading channel. The dashed line represents an approximation to the BER using the union bound, of which only the first 4 terms are used.

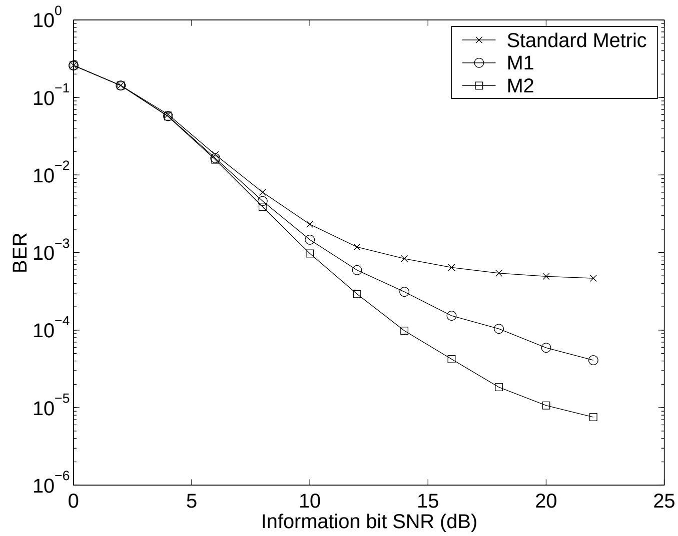

Fig. 4. Comparison of modified and standard VD metrics for 4 users with $\grave{N} = 16$ using a half-rate, memory order 2, code and soft-decision decoding with the ICUD scheme on a flat Rayleigh fading channel.

information bit SNR $\grave{\gamma}$, which is equal to 2γ for the half-rate encoders used, since $\grave{\gamma}$ is independent of the code rate used. The SNR is equal for all users. The dashed line in Fig. 3 is the BER calculated by applying the so-called "standard Gaussian approximation" [12], in which the RMAI is modelled as a Gaussian random variable. All other curves are obtained using computer simulations. Fig. 4 compares the performance obtained using the standard Euclidean metric to the performance using the modified metrics M1 and M2 for a 4-user system with $\grave{N} = 16$ on a frequency nonselective Rayleigh fading channel. From the aforementioned two figures, we see that the BER improvement due to the modified metrics is roughly one order of magnitude. As the number of users and spreading factor increase while their ratio remains unchanged the performance gap between the M1 and M2 metrics is expected to decrease. This is because the difference between the unconditional MAI variance of (7) and the conditional MAI variance of (8) will decrease based on the CLT.

It is worth mentioning that, at low SNR, the inaccuracy in the estimate of the variance in (6) limits the improvement of the modified metrics, which is not the case at high SNR. This can be easily explained for the case of equal user energies and the M1 metric. In that case, when σ^2 is negligible (i.e. at high SNR), $\psi_{1,i}$ consists of the product of a constant factor and a time-varying factor $\sum_{j=2}^{K} r_{1j,i}^2$, which can be calculated exactly. Only the time-varying factor affects the path selection since any constant in the denominator of (5) will appear in both operands of the compare operation of the VD and is therefore irrelevant. Hence, any inaccuracy in the estimate of (6) has no effect on performance. At low SNR, on the other hand, where σ^2 cannot be neglected, decomposing $\psi_{1,i}$ into time-varying and constant factors is not possible and the exact estimate is needed. A similar effect is observed with the modified metric M2 although $\psi_{1,i}$ cannot be written as the product of a constant and time-varying factor in the case of M2. is not composed of the product of a This problem at low SNR can be mitigated by multiplying the RMAI variance term, i.e. the summation, in (6) by a scale factor, say α, that relates the true variance to the variance estimated from (6). In other words, α would partly compensate for the dependencies we ignored in estimating $\psi_{1,i}$. The factor α would probably be obtained empirically and may be a function of fixed system parameters such as N for fixed-rate systems, or parameters that change infrequently such as K, N for multi-rate systems, and the SNR.

B. Improved VD Metrics for PDIC Structures

The basic idea for modifying the VD branch metric is the same for PDIC structures. In estimating $\psi_{1,i}$, however, we ignore the dependence between $e_{j,i}$ and $|c_{j,i}|$ in the RMAI terms. This is because it is weaker than in the ICUD case and much more difficult to take into account since the first stage errors are due to error events from the first VD bank. With this simplifying assumption it can be easily shown that $\psi_{1,i}$ may be approximated by

$$\begin{aligned} \psi_{1,i} &\approx 4 \sum_{j=2}^{K} r_{1j,i}^2 \ \mathbf{E}[\beta_{j1,i}^2] \ \mathbf{E}[|c_j|^2] \ \mathbf{E}[e_j^2] \ + \ \sigma^2 \\ &= 4 \sum_{j=2}^{K} r_{1j,i}^2 \ \mathcal{E}_j \ P_j \ + \ \sigma^2 \qquad (9) \end{aligned}$$

where P_j is the first stage code bit error probability. Strictly speaking, P_j should be a function of the crosscorrelation values on which we condition. However, to simplify the metric we use the average value of the code BER for each user, which we upper-bound using a union bound and the exact expression for the pairwise error probability [13].

Fig. 5 shows the performance improvement due to the modified metric based on (9), which we denote by M3. At high

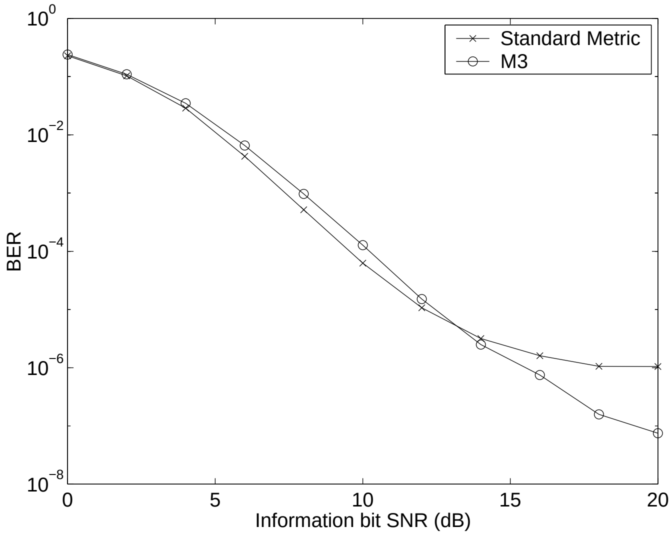

Fig. 5. Comparison of modified and standard VD metrics for 6 users with $\grave{N} = 32$ using a half-rate, memory order 2, code and soft-decision decoding with the PDIC scheme on a flat Rayleigh fading channel.

SNR, the BER is one order of magnitude lower when using the M3 metric. Similar to the ICUD case, performance at low to moderate SNR suffers from the inaccuracy of the estimate of $\psi_{1,i}$. Indeed, up to a certain SNR the performance using the M3 metric is even worse than the standard Euclidean branch metric. This is because of the crude estimate used in this case for the RMAI variance conditioned on the crosscorrelations in addition to the looseness of the union bound at low SNR which results in a poor estimate of P_j. Thus, performance in a practical system, which could use measurements of P_j during training, should be better. Furthermore, and similar to the ICUD case, the performance gain may be increased at low SNR by more realistic estimates of the RMAI variance. Such better estimates can be obtained from (9) by inclusion of a scale factor multiplying the summation as described for the ICUD case.

IV. Discussion and Conclusions

We considered subtractive IC receiver structures for CDMA systems with forward error correction. Single-user VDs are usually employed in multiuser receivers for their relatively low complexity. Despite many proposals for multiuser detection and single-user decoding, to the authors' knowledge little attention if any has been given to improving the single-user VD branch metrics in multiuser receivers. We proposed several modifications of varying complexity and performance to the VD branch metrics for CDMA systems with long sequences, such as IS-95 [7]. The modifications were based on modelling the RMAI as Gaussian conditioned on the time-varying user crosscorrelations and deriving an accurate estimate of the conditional RMAI variance. Significant performance improvement was demonstrated at moderate to high SNR. It should be mentioned that the modified VDs require an estimate of the average SNR or, equivalently, the noise power except at high SNR where the noise variance σ^2 may be neglected in (6). This information is not explicitly needed for the IC stage, but is not difficult to calculate.

The performance gain may be further increased based on more accurate estimates of the conditional variance of the RMAI plus noise, particularly for the low SNR region. This improved accuracy may be obtained by inclusion of scale factors (to be obtained experimentally) in the derived expressions. The metrics were proposed for two alternative receiver structures: ICUD, in which IC occurs before any decoding takes place, and PDIC, where decoded decisions are used in IC. The performance gain due to the modified metrics was more significant for the ICUD structure than for the PDIC structure. This is partly due to the higher accuracy of the RMAI variance estimate for ICUD schemes and also possibly due to the difference in the mechanism of error occurrence between the two schemes.

We conclude by pointing out that the single-user VD branch metrics may be further improved, generally speaking, by incorporating additional knowledge about the interfering users' signal parameters. This may be very attractive particularly when this information is readily available or must be acquired for the IC stages of the receiver.

Appendix I
Proof of Equation (6)

We first seek to obtain $\mathbf{Var}[\zeta_j]$ conditioned on $\{r_{1j}\}$, where we omit the subscript i denoting time-dependence for convenience. The conditional variance may be evaluated by $\mathbf{E}[\zeta_j^2] = \mathbf{E}[(2r_{1j}|c_j|\beta_{j1}e_j)^2]$ since $\mathbf{E}[\zeta_j] = 0$ from symmetry arguments. Each of $\beta_{j1}, |c_j|$, and r_{1j} is pairwise dependent with e_j. However, these dependencies are not very strong and can be ignored except for the dependence between $|c_j|$ and e_j. Thus, we approximate the desired variance by

$$\mathbf{Var}[\zeta_j] \approx 4\ r_{1j}^2\ \mathbf{E}[\beta_{j1}^2]\ \mathbf{E}[(|c_j|e_j)^2] \qquad (10)$$

From basic probability theory we can prove that the probability density function of β_{jk} for all j and k is given by $f_\beta(x) = \frac{1}{\pi\sqrt{1-x^2}},\ \ -1 < x < 1$, and it is simple to show that $\mathbf{E}[\beta_{jk}^2] = 1/2$. We are now left with the last term on the right-hand side of the variance approximation, which we evaluate as follows:

$$\mathbf{E}\left[|c_j|^2 e_j^2\right] = \mathbf{E}_{|c_j|}\left(|c_j|^2 \mathbf{E}\left[e_j^2 \middle| |c_j|\right]\right) \qquad (11)$$

The conditional expectation can be expressed as $\mathbf{E}\left[e_j^2 \middle| |c_j|\right] = \sum_{y\in\{-1,0,1\}} y^2 p(y|x) = p(-1|x) + p(1|x) = 2p(-1|x)$ where $p(y|x) \triangleq p_{e_j||c_j|}(y|x)$ is the conditional probability mass function of e_j given $|c_j|$, and the last step is due to problem symmetry. Now, $p(-1|x) = Pr(b_j = -1, \hat{b}_j = 1|\ x) = Pr(\hat{b}_j = 1|\ x, b_j = -1)\ Pr(b_j = -1) = \frac{1}{2}Q(x/\eta_j)$ where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-t^2/2}dt$, and $\eta_j^2 = \sum_{l\neq j}\mathcal{E}_l(1/N) + \sigma^2$ is the variance of the MAI seen by user j. Substituting this result back in (11), we may write $\mathbf{E}\left[|c_j|^2 e_j^2\right] = \int_0^\infty x^2\ Q(x/\eta_j)\ f_{|c_j|}(x)\ dx$ where $f_{|c_j|}(x) = \frac{x}{\alpha^2}\ e^{-x^2/2\alpha^2}$ is the Rayleigh PDF of $|c_j|$ and $\alpha^2 = \mathcal{E}_j = \gamma_j\sigma^2$. Applying the change of variable $x^2 = A$, the right-hand side in the above equations can be expressed as

$$\int_0^\infty A \, \frac{e^{-A/2\alpha^2}}{2\alpha^2} \, Q(\sqrt{A/\eta_j^2}) \, dA = \int_0^\infty \underbrace{\left(\frac{1}{\sqrt{2\pi}}\int_{\sqrt{A/\eta_j^2}}^\infty e^{-t^2/2}\,dt\right)}_{U} \underbrace{\frac{Ae^{-A/2\alpha^2}}{2\alpha^2}\,dA}_{dV}$$

Evaluating the above integration by parts, $\int_0^\infty U \; dV = UV\big|_0^\infty - \int_0^\infty V dU$ where U and dV are as indicated. We use Leibnitz's theorem for differentiation of an integral to evaluate dU, perform a second integration by parts and after some manipulations, arrive at $\mathbf{E}\left[|c_j|^2 e_j^2\right] = \frac{\alpha^2}{2} - \frac{1}{2\eta_j\sqrt{2\pi}}\left[\int_0^\infty \left(A^{1/2}e^{-cA} + \alpha^2 A^{-1/2} e^{-cA}\right) dA\right]$ where $c = \frac{2\eta_j^2+\alpha^2}{2\eta_j^2\alpha^2}$. Using the integral form of the Gamma-function $\int_0^\infty t^{n-1}e^{-at}dt = \Gamma(n)/a^n$, where $a, n > 0$, we obtain $\mathbf{E}\left[|c_j|^2 e_j^2\right] = \frac{\alpha^2}{2} - \frac{1}{2\eta_j\sqrt{2\pi}}\left[\frac{\Gamma(3/2)}{c^{3/2}} + \alpha^2\frac{\Gamma(1/2)}{c^{1/2}}\right]$. Using $\Gamma(3/2) = \sqrt{\pi}/2$ and $\Gamma(1/2) = \sqrt{\pi}$, and rewriting c and α in terms of the system parameters, and after some manipulation we arrive at $\mathbf{E}[(|c_j|e_j)^2] = \mathcal{E}_j\left[1 - \sqrt{\frac{\mathcal{E}_j}{\mathcal{E}_j+\eta_j^2}}\left(\frac{3\eta_j^2+2\mathcal{E}_j}{2\eta_j^2+2\mathcal{E}_j}\right)\right]$. Substituting this term in the approximate formula for the desired variance (10) and approximating the variance of the total RMAI as the sum of the variances of the individual RMAI terms, we arrive at (6).

REFERENCES

[1] T. R. Giallorenzi and S. G. Wilson, "Multiuser ML Sequence Estimator for Convolutionally Coded Asynchronous DS-CDMA Systems," *IEEE Trans. on Comm.*, vol. 44, pp. 997–1008, August 1996.

[2] P. Hoeher, "On Channel Coding and Multiuser Detection for DS-CDMA," in *Proceedings of the 2nd International Conference on Universal Personal Communications, Ottawa, Canada*, pp. 641–646, October 1993.

[3] A. Lampe and J. B. Huber, "Iterative Interference Cancellation for DS-CDMA Systems with High System Loads Using Reliability-dependent Feedback," *IEEE Transactions on Vehicular Technology*, vol. 51, pp. 445–452, May 2002.

[4] A. J. Viterbi, "Very Low Rate Convolutional Codes for Maximum Theoretical Performance of Spread-Spectrum Multiple-Access Channels," *IEEE Journal on Selected Areas in Communications*, vol. 8, pp. 641–649, May 1990.

[5] T. R. Giallorenzi and S. G. Wilson, "Suboptimum Multiuser Receivers for Convolutionally Coded Asynchronous DS-CDMA Systems," *IEEE Trans. on Comm.*, vol. 44, pp. 1183–1196, September 1996.

[6] M. K. Varanasi and B. Aazhang, "Multistage Detection in Asynchronous Code-Division Multiple-Access Communications," *IEEE Trans. on Comm.*, vol. COM-38, pp. 509–519, April 1990.

[7] TIA/EIA-95-B, "Mobile Station-Base Station Compatibility Standard for Dual-Mode Spread Spectrum Systems," 1998.

[8] A. Elezabi and A. Duel-Hallen, "Improved Single-User Decoder Metrics for Two-Stage Detectors in DS-CDMA," in *IEEE Proc. of Vehic. Tech Conf.*, pp. 1276–1281, Sept. 2000.

[9] A. Elezabi and A. Duel-Hallen, "Two-Stage Receiver Structures for Coded CDMA Systems," in *IEEE Vehicular Technology Conference*, pp. 1425–1429, May 1999.

[10] R. J. Serfling, "Contributions to Central Limit Theory for Dependent Variables," *Annals of Math. Stat.*, vol. 39, no. 4, pp. 1158–1175, 1968.

[11] W. Zeng and J. Moon, "Modified Viterbi Detection for Recording Channels with Jitter," in *Proceedings of the IEEE International Communications Conference*, pp. 131–135, June 1992.

[12] M. Pursley, "Performance Evaluation for Phase-Coded Spread-Spectrum Multiple-Access Communication-Part I: System Analysis," *IEEE Trans. on Comm.*, vol. COM-25, pp. 795–799, August 1977.

[13] A. Elezabi and A. Duel-Hallen, "Conventional-based Two-Stage Detectors in Coded CDMA," in *Proc. of IEEE Int. Symp. on Info. Th.*, p. 282, Aug. 1998.

PLACE PHOTO HERE

Ayman Elezabi Ayman Elezabi received his B.Sc. degree in Electrical Engineering from Cairo University in 1989, and his M.Sc. and Ph.D. in Electrical and Computer Engineering from North Carolina State University in 1995 and 2000, respectively. He worked at BOPS Inc. between 2000 and 2002 on wireless networks and ADSL. Prior to and during his graduate studies, he worked at the IBM Cairo Scientific Center on text-to-speech systems and at Analog Devices, Greensboro, NC and Ericsson, RTP, NC on CDMA systems. He is now a visiting assistant professor in the Electronics Engineering Department at the American University in Cairo. He is also a consultant at the Engineering Office for Integrated Projects in Cairo, Egypt. His current research interests are in wireless communications, channel coding, and physical layer communications in general.

PLACE PHOTO HERE

Alexandra Duel-Hallen Alexandra Duel-Hallen received B.S. degree in mathematics from Case Western Reserve University in 1982, M.S. degree in Computer, Information and Control Engineering from the University of Michigan in 1983, and Ph.D. in Electrical Engineering from Cornell University in 1987. During 1987-1990 she was a Visiting Assistant Professor at the School of Electrical Engineering, Cornell University, Ithaca, NY. In 1990-1992, she was with the Mathematical Sciences Research Center, AT&T Bell Laboratories, Murray Hill, NJ. She is an Associate Professor at the Department of Electrical and Computer Engineering at North Carolina State University, Raleigh, NC, which she joined in January 1993. From 1990 to1996, Dr. Duel-Hallen was Editor for Communication Theory for the IEEE Transactions on Communications. During 2000- 2002, she has served as Guest Editor of two Special Issues on Multiuser Detection for the IEEE Journal on Selected Areas in Communications. Dr. Duel-Hallen's current research interests are in wireless and multiuser communications. Her 1993 paper was selected for the IEEE Communications Society 50th Anniversary Journal Collection as one of 41 key papers in physical and link layer areas, 1952-2002.